CHAPTER 3 -MATRICES

CAPSULE 1

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DEFINITION

- A Matrix is an ordered rectangular array of numbers or functions.The numbers or functions are called the elements or the entries of the matrix.
- We denote matrices by capital letters

Eg: $A = \begin{array}{cc} \begin{bmatrix} -2 & 5 \\ 0 & \sqrt{5} \\ 3 & 6 \end{bmatrix} & \begin{array}{l} \text{R1} \\ \text{R2} \\ \text{R3} \end{array} \\ \begin{array}{cc} \text{C1} & \text{C2} \end{array} & \end{array}$

3 X 2 MATRIX

Eg: B = $\begin{pmatrix} 2 & 1/2 & 5 \\ a & -3 & 0.5 \end{pmatrix}$ **2 x 3 Matrix**

GENERAL FORM

- A matrix having m rows and n columns is called a matrix of order m x n
- In general , an m x n matrix has the following rectangular array:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

- Compact form the above matrix is represented by $[a_{ij}]_{m \times n}$ or $A = [a_{ij}]. 1 \leq i \leq m, 1 \leq j \leq n ; i, j \in N$

EXAMPLES

- Eg: P = $\begin{bmatrix} 1 & 2 & 3 \\ -4 & 1 & 7 \\ 0 & 6 & 1/2 \end{bmatrix}$ order 3 x 3
- Eg: Q = $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ order 3 x 1
- Eg: R = $\begin{bmatrix} a & x & 5 \end{bmatrix}$ order 1 x 3
- Eg: T = $\begin{pmatrix} \frac{1}{2} \\ 0.2 \end{pmatrix}$ order 2 x 1
- Eg: B = $\begin{pmatrix} 4 & 100 \\ 14 & 1234 \end{pmatrix}$ order 2 x 2

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DIFFERENT TYPES OF MATRICES

Types of Matrices

- **Row Matrix:** A matrix having only one row and any number of columns is called a row matrix. Eg: R = $\begin{bmatrix} a & x & 5 \end{bmatrix}$
- **Column Matrix :**A matrix having only one column and any number of rows is called column matrix. Eg: Q = $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$
- **Square Matrix** : A matrix in which the number of rows are equal to number of columns is said to be a square matrix. So, m = n. Eg: B = $\begin{pmatrix} 4 & 1 \\ 9 & 3 \end{pmatrix}$; C == $\begin{bmatrix} 1 & 2 & 3 \\ -4 & 1 & 7 \\ 0 & 6 & 1/2 \end{bmatrix}$
- **Diagonal Matrix :**A square matrix A = $[a_{ij}]_{m \times n}$, is called a diagonal matrix, ifall the elements except those in the leading diagonals are zero. Eg: A = $\begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix}$; P = = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$

CONTINUE…

- **Scalar Matrix :**A Diagonal matrix in which all diagonal elements are equal, is called scalar matrix.Eg: P = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
- **Unit/Identity Matrix :**A square matrix, in which every non-diagonal element is zero and every diagonal element is 1, is called, **unit matrix** or an **identity matrix.** Eg: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} : (1) : \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- **Zero Matrix(Null Matrix):** A matrix is said to be zero matrix if all its elements are zero. We denote zero matrix by **O**

  Eg: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$; $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$; $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

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EQUALITY OF MATRICES

- **Equal Matrices**

Two matrices A and B are said to be equal if ,

i) both having same order and

ii) corresponding elements of the matrices are equal.

- $A = \begin{bmatrix} 1 & b & 3 \\ x & 1 & 7 \\ 0 & c & 1/2 \end{bmatrix}$ and $B = \begin{bmatrix} a & 2 & 3 \\ -4 & 1 & y \\ z & 6 & 1/2 \end{bmatrix}$ are equal , then a = 1: b = 2: x = -4

  y = 7: c = 6; z = 0

OPERATIONS ON MATRICES

- **Addition of Matrices**: The sum of Two matrices A and B is defined , only if A and B are of the same order. Then add the corresponding elements.

Eg 1 : $A = \begin{pmatrix} 4 & -1 \\ -9 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 1 \\ 7 & -5 \end{pmatrix}$

$$A + B = \begin{pmatrix} 4 & -1 \\ -9 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 7 & -5 \end{pmatrix} = \begin{pmatrix} 4+2 & -1+1 \\ -9+7 & 3+-5 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ -2 & -2 \end{pmatrix}$$

Eg 2 : $P = \begin{pmatrix} -7 & 1/2 & 5 \\ 6 & -3 & 0.5 \end{pmatrix}$ $Q = \begin{pmatrix} 2 & 3/2 & 0 \\ a & -3 & 1.5 \end{pmatrix}$

$$P + Q = \begin{pmatrix} -7 & 1/2 & 5 \\ 6 & -3 & 0.5 \end{pmatrix} + \begin{pmatrix} 2 & 3/2 & 0 \\ a & -3 & 1.5 \end{pmatrix} = \begin{pmatrix} -5 & 2 & 5 \\ 6+a & -6 & 2 \end{pmatrix}$$

CONTINUE….

- **Difference of Matrices:** If A and B are two matrices of same order, then

  A – B = A + (-B)

Eg: A = $\begin{pmatrix} 4 & -1 \\ -9 & 3 \end{pmatrix}$ B = $\begin{pmatrix} 2 & 1 \\ 7 & -5 \end{pmatrix}$

$$A - B = \begin{pmatrix} 4 & -1 \\ -9 & 3 \end{pmatrix} + \begin{pmatrix} -2 & -1 \\ -7 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ -16 & 8 \end{pmatrix}$$

- **Multiplication of a Matrix by a Scalar:**

  Eg: $A = \begin{bmatrix} 1 & b & 3 \\ x & 1 & 7 \\ 0 & c & 1/2 \end{bmatrix}$ **Multiply matrix A by a scalar 2, then it will be 2A**

2A = $\begin{bmatrix} 2 & 2b & 6 \\ 2x & 2 & 14 \\ 0 & 2c & 1 \end{bmatrix}$

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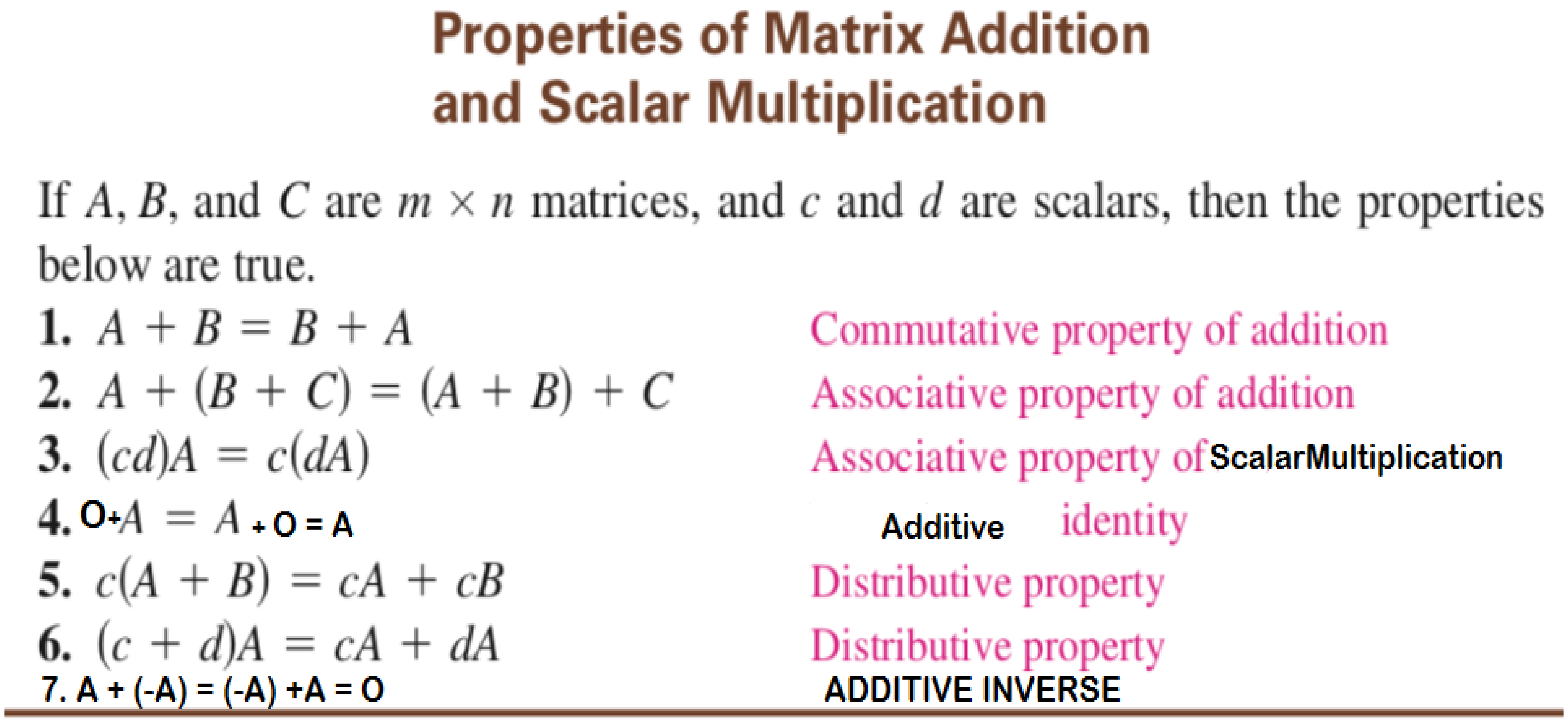
PROPERTIES



Properties of Matrix Addition and Scalar Multiplication

If A, B, and C are $m \times n$ matrices, and c and d are scalars, then the properties below are true.

1. $A + B = B + A$	Commutative property of addition
2. $A + (B + C) = (A + B) + C$	Associative property of addition
3. $(cd)A = c(dA)$	Associative property of ScalarMultiplication
4. $O + A = A + O = A$	Additive identity
5. $c(A + B) = cA + cB$	Distributive property
6. $(c + d)A = cA + dA$	Distributive property
7. $A + (-A) = (-A) + A = O$	ADDITIVE INVERSE

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RELATED QUESTIONS

- 1)Construct a 2 x 3[ ] matrix whose elements are given by a_{ij} = 2i – j

Then A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$: a_{11} = 2x1-1 =1; a_{12} = 2 x 1 – 2 = 0 ; a_{13} = 2 x 1 – 3 = -1

a_{21} = 2 x 2 – 1=3 ; a_{22} = 2 x 2 – 2 =2 ; a_{23} = 2 x 2 – 3 = 1

A = $\begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$

- 2) .A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ C = $\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

a) find A + B – 2C

$\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - 2\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}+\begin{bmatrix} 4 & -10 \\ -6 & -8 \end{bmatrix}= \begin{bmatrix} 7 & -3 \\ -5 & -1 \end{bmatrix}$

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QUESTION 3

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- Find x , y , z if $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Ans: $x+y+z = 9$……..(1)

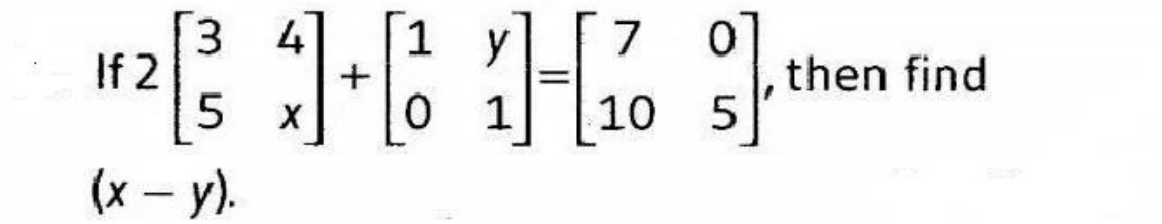
$y+z$ = 7………(2)

$x+z$ = 5 ………(3)

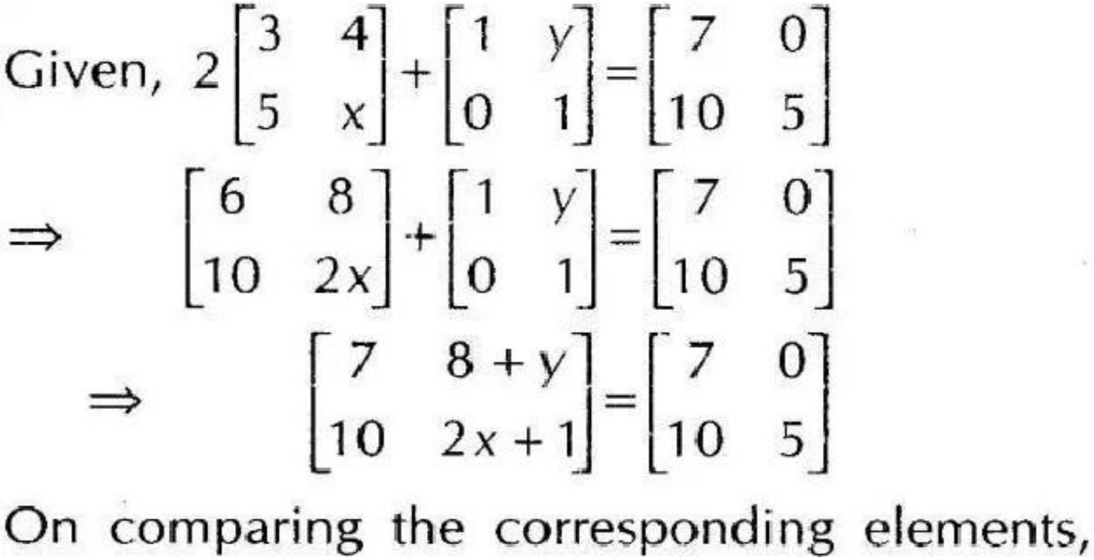
Substitute (2) in (1) → $x + 7 = 9$ → x = 2

Substitute x in (3) & (1) → z = 3 , y = 4

QUESTION 4



If $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$, then find $(x-y)$.



Given, $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}6 & 8\\10 & 2x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}7 & 8+y\\10 & 2x+1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$$

On comparing the corresponding elements,

2x+1 =5 ; x = 2
8 + y = 0 : y = -8

; x - y = 2-(-8) =10

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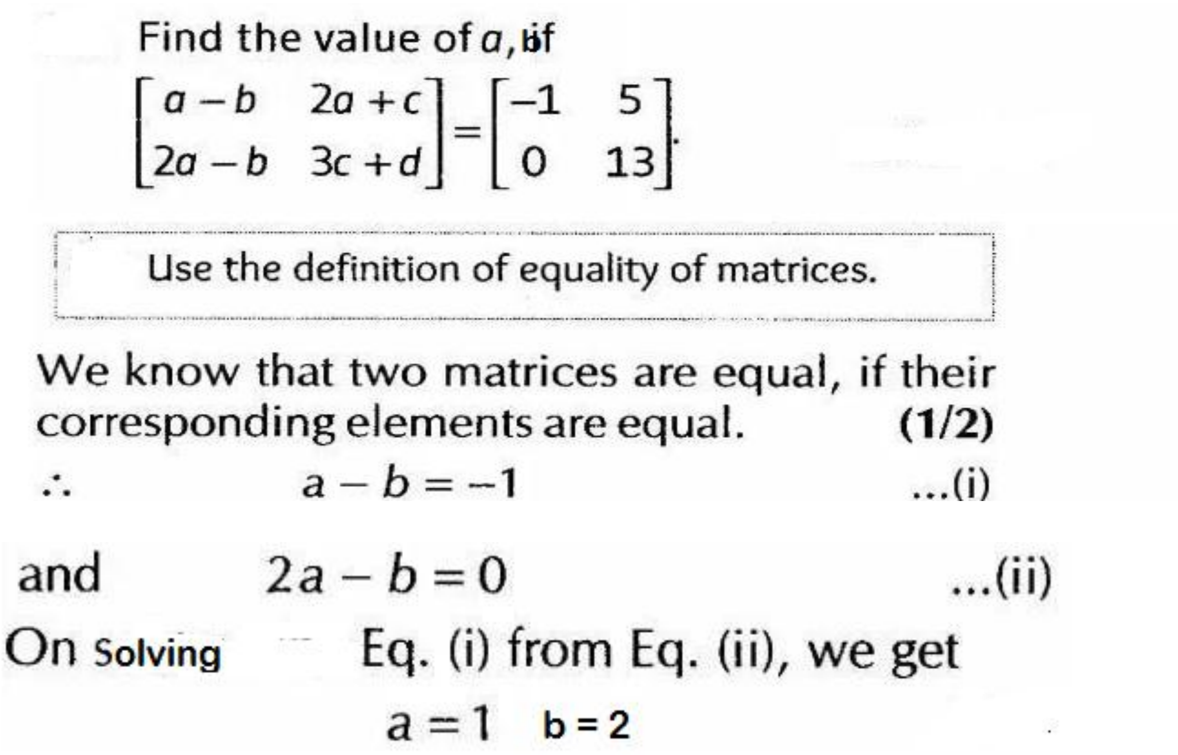
QUESTION 5



Find the value of a, b if

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}.$$

Use the definition of equality of matrices.

We know that two matrices are equal, if their corresponding elements are equal. **(1/2)**

$\therefore \quad a - b = -1 \quad \ldots(i)$

and $\quad 2a - b = 0 \quad \ldots(ii)$

On Solving Eq. (i) from Eq. (ii), we get

$a = 1 \quad b = 2$

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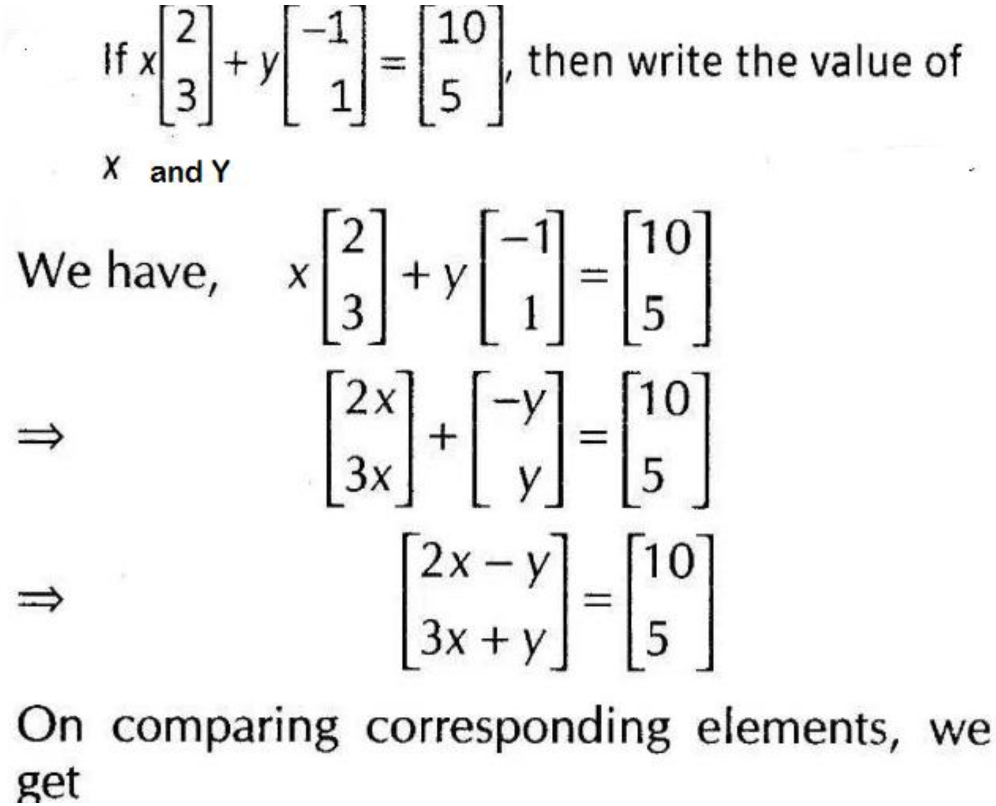
QUESTION 6



If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$, then write the value of

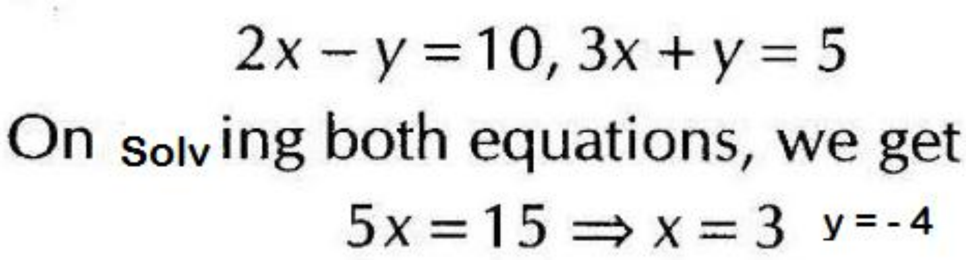
x and Y

We have, $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2x-y\\3x+y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$

On comparing corresponding elements, we get



$$2x-y=10,\ 3x+y=5$$

On Solving both equations, we get

$$5x=15 \Rightarrow x=3 \quad y=-4$$

QUESTION 7

- Find **X** and **Y** , **if** $2X + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix}$ $3X + 2Y = \begin{pmatrix} 2 & -2 \\ -1 & 5 \end{pmatrix}$

$2X + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix}$..........(1) & $3X + 2Y = \begin{pmatrix} 2 & -2 \\ -1 & 5 \end{pmatrix}$ ....(2)

(1) x 2 → $4X + 6Y = \begin{pmatrix} 4 & 6 \\ 8 & 0 \end{pmatrix}$................(3)

(2) x 3 → $9X + 6Y = \begin{pmatrix} 6 & -6 \\ -3 & 15 \end{pmatrix}$.............(4) Solving (3) and (4)

(4) – (3) → $5X = \begin{pmatrix} 2 & -12 \\ -11 & 15 \end{pmatrix}$ → $X = \begin{pmatrix} 2/5 & -12/5 \\ -11/5 & 3 \end{pmatrix}$

Substitute in (1) or (2) and get $Y = \begin{pmatrix} 2/5 & 13/5 \\ 14/5 & -2 \end{pmatrix}$

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CHAPTER -3: MATRICES

PART – 2

TOPIC: MULTIPLICATION OF MATRICES

Condition for Matrix Multiplication

- The Product of Two Matrices A and B is defined if the number of columns of A is equal to the number of rows of B

$$[A]_{m\,x\,\underset{\bigstar}{n}} \quad X \quad [B]_{\underset{\bigstar}{n}\,x\,p} \quad = \quad [AB]_{m\,x\,P}$$

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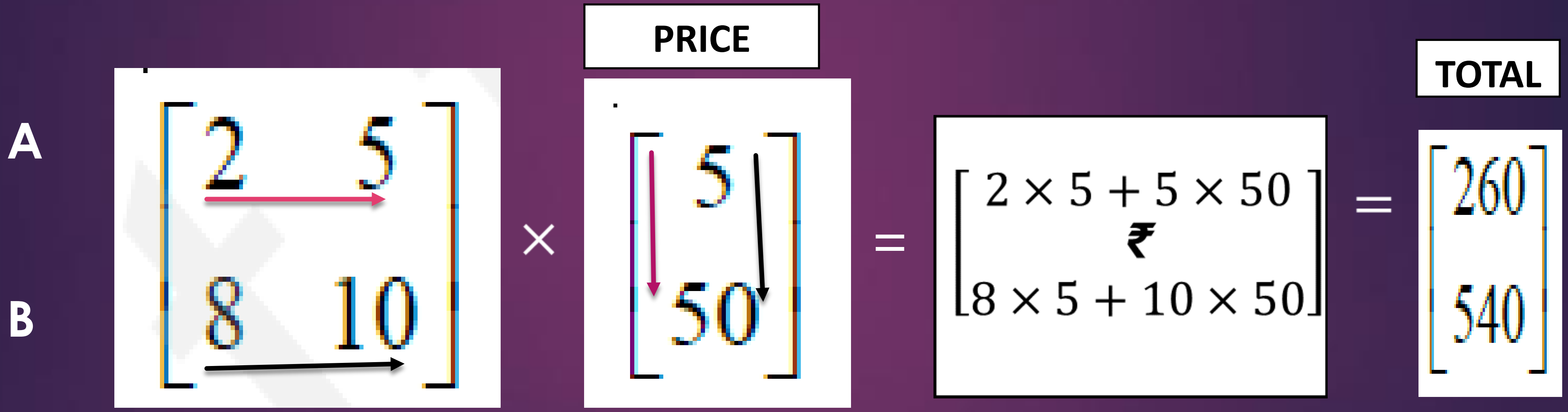
Multiplication of matrices:



$$\begin{array}{c} \\ A \\ \\ B \end{array} \begin{bmatrix} 2 & 5 \\ 8 & 10 \end{bmatrix} \times \begin{array}{c} \text{PRICE} \\ \begin{bmatrix} 5 \\ 50 \end{bmatrix} \end{array} = \begin{bmatrix} 2 \times 5 + 5 \times 50 \\ \text{₹} \\ 8 \times 5 + 10 \times 50 \end{bmatrix} = \begin{array}{c} \text{TOTAL} \\ \begin{bmatrix} 260 \\ 540 \end{bmatrix} \end{array}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 2 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 10 & 12 \\ 5 & 4 \\ 3 & 2 \end{bmatrix}, \textit{Find } AB$$

$$\begin{bmatrix} R_1C_1 & R_1C_2 & R_1C_3 \\ R_2C_1 & R_2C_2 & R_2C_3 \\ R_3C_1 & R_3C_2 & R_3C_3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 2 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 10 & 12 \\ 5 & 4 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 10 + 2 \times 5 + 3 \times 3 & 1 \times 12 + 2 \times 4 + 3 \times 2 \\ 0 \times 10 + 2 \times 5 + 4 \times 3 & 0 \times 12 + 2 \times 4 + 4 \times 2 \\ 2 \times 10 + 2 \times 5 + 1 \times 3 & 2 \times 12 + 2 \times 4 + 1 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & 26 \\ 22 & 16 \\ 33 & 34 \end{bmatrix}$$

NOTE: $A_{3\times3} \times B_{3\times2} = AB_{3\times2}$

$$AB = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$$

$$\boldsymbol{BA} = \begin{bmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{4} & \mathbf{5} \\ \mathbf{2} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{-2} & \mathbf{3} \\ \mathbf{-4} & \mathbf{2} & \mathbf{5} \end{bmatrix} = \begin{bmatrix} \mathbf{-10} & \mathbf{2} & \mathbf{21} \\ \mathbf{-16} & \mathbf{2} & \mathbf{37} \\ \mathbf{-2} & \mathbf{-2} & \mathbf{11} \end{bmatrix}$$

$$AB \neq BA$$

PROPERTIES OF MULTIPLICATION

1. $AB \neq BA$

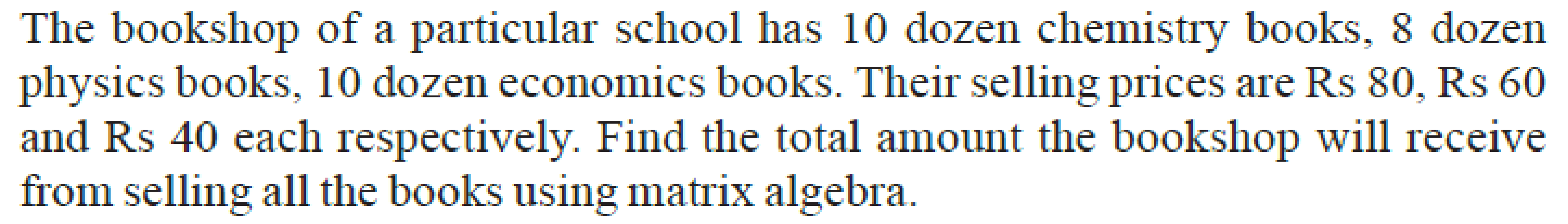
2. $If\ A_{m\times n} and\ B\ _{n\times p}, then\ AB_{m\times p}$

3. $If\ A\ is\ a\ square\ matrix, A \times I = I \times A = A$

4. $A(BC) = (AB)C$, WHENEVER EQUALITY OF BOTH SIDES ARE DEFINED

5. $A(B + C) = AB + AC$ AND $(A + B)C = AC + BC$

WHENEVER EQUALITY OF BOTH SIDES ARE DEFINED



The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs 80, Rs 60 and Rs 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

$$\mathbf{12} \times [\mathbf{10} \quad \mathbf{8} \quad \mathbf{10}] \times \begin{bmatrix}\mathbf{80}\\ \mathbf{60}\\ \mathbf{40}\end{bmatrix} = \mathbf{12}[\mathbf{800} + \mathbf{480} + \mathbf{400}] = [20160]$$

$$A_{1\times 3} \times B_{3\times 1} = AB_{1\times 1}$$

Eg:$If\ A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}, then\ find\ AB\ and\ BA$

$(A)_{1X3} \cdot (B)_{3X1} = (AB)_{1X1}$

ANS: $$\boldsymbol{AB} = \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \end{pmatrix} \begin{pmatrix} \mathbf{0} \\ \mathbf{-2} \\ \mathbf{3} \end{pmatrix} = (\mathbf{0} + (\mathbf{-4}) + \mathbf{9}) = (\mathbf{5})$$

$$\mathbf{BA} = \begin{pmatrix} \mathbf{0} \\ \mathbf{-2} \\ \mathbf{3} \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{-2} & \mathbf{-4} & \mathbf{-6} \\ \mathbf{3} & \mathbf{6} & \mathbf{9} \end{pmatrix}$$

$(B)_{3X1} \cdot (A)_{1X3} = (BA)_{3X3}$

Multiplication of matrices:

Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$

$$X_{2\times 2}\times Y_{2\times 3}=z_{2\times 3}$$

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{pmatrix}=\begin{pmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{pmatrix}$$

$$\begin{pmatrix}a+4b & 2a+5b & 3a+6b\\ c+4d & 2c+5d & 3c+6d\end{pmatrix}=\begin{pmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{pmatrix}$$

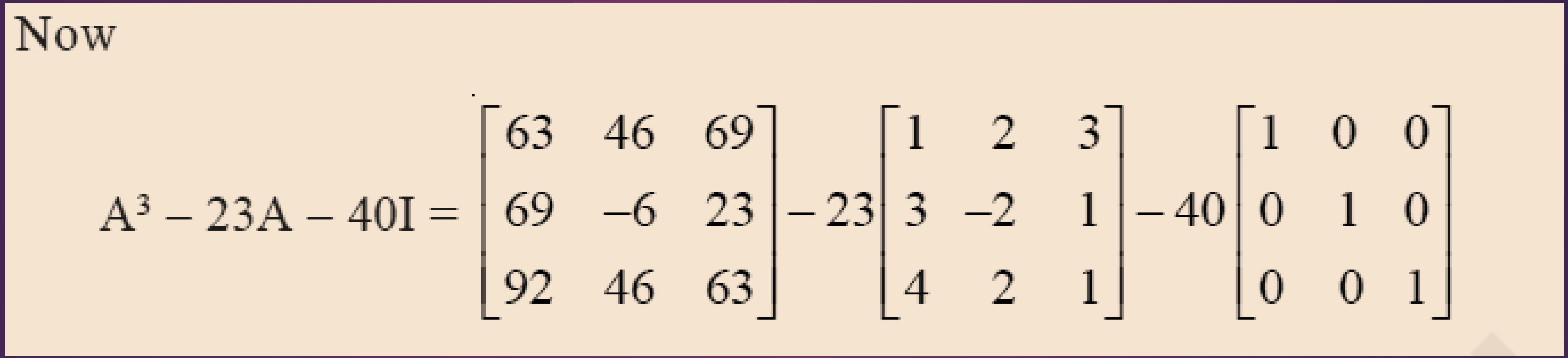
solving linear equations, we get a = 1, b= -2, c= 2 and d= 0

Hence X= $\begin{pmatrix}1 & -2\\ 2 & 0\end{pmatrix}$

Multiplication of matrices:

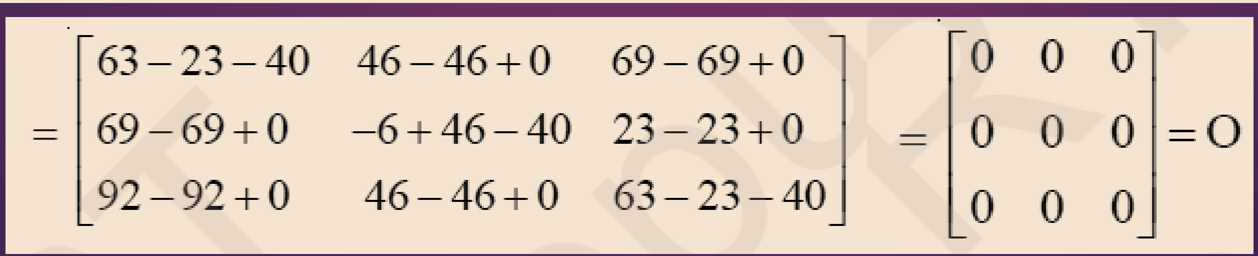
If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & -1 \\ 4 & 2 & 1 \end{bmatrix}$ *Then Prove that* $A^3 - 23A - 40I = O$

What is A^3 = A.A.A



Now

$$A^3 - 23A - 40I = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



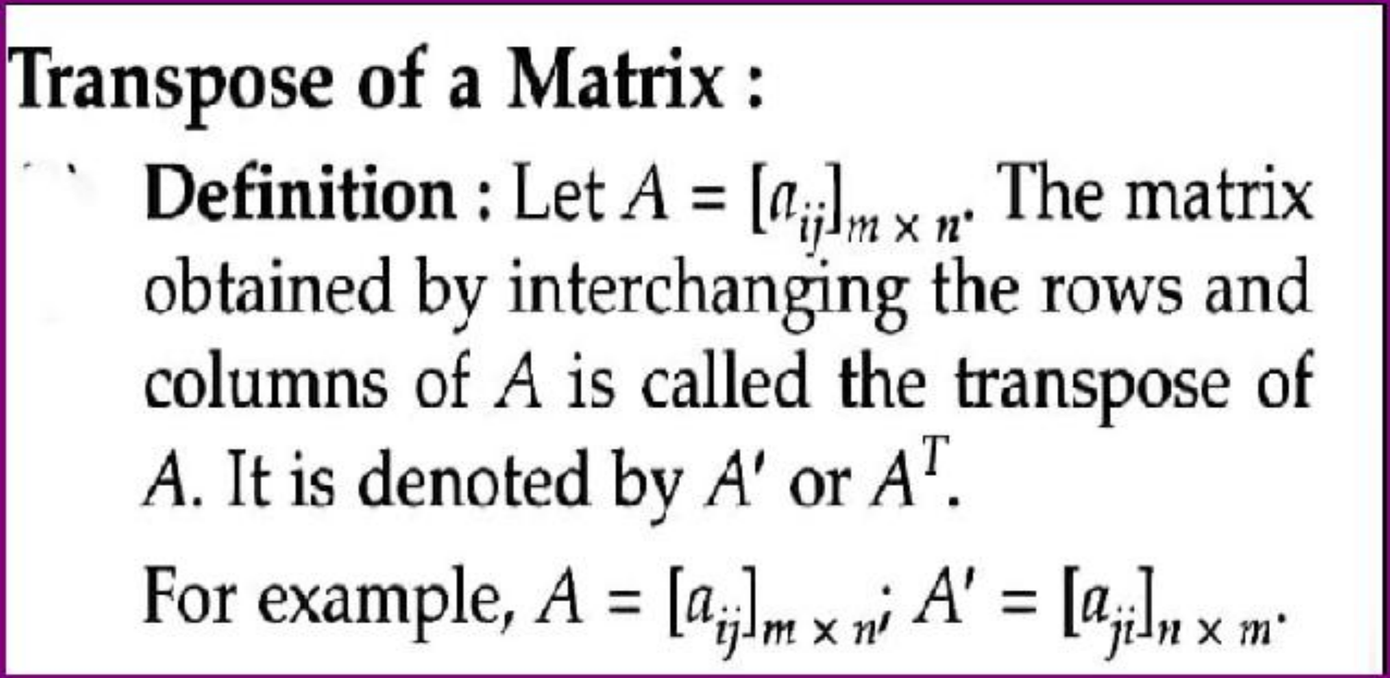
$$= \begin{bmatrix} 63-23-40 & 46-46+0 & 69-69+0 \\ 69-69+0 & -6+46-40 & 23-23+0 \\ 92-92+0 & 46-46+0 & 63-23-40 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

MATRICES –PART 3

TRANSPOSE OF A MATRIX ,SYMMETRIC MATRIX AND SKEW SYMMETRIC MATRIX

TESSY ROY VARGHESE

TRANSPOSE OF A MATRIX

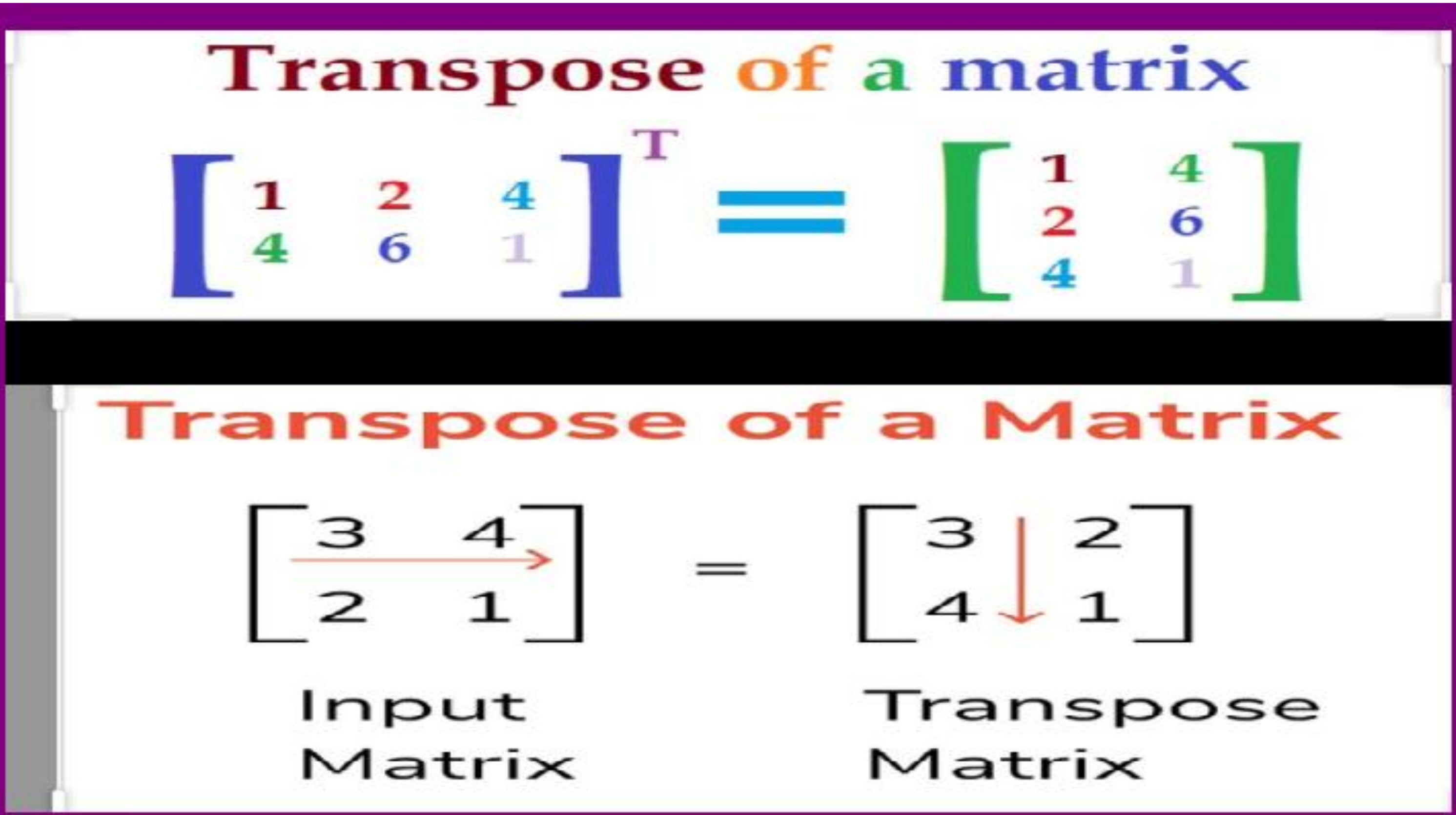


Transpose of a Matrix :

Definition : Let $A = [a_{ij}]_{m \times n}$. The matrix obtained by interchanging the rows and columns of A is called the transpose of A. It is denoted by A' or A^T.

For example, $A = [a_{ij}]_{m \times n}$; $A' = [a_{ji}]_{n \times m}$.

Eg:



Transpose of a matrix

$$\begin{bmatrix} 1 & 2 & 4 \\ 4 & 6 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 6 \\ 4 & 1 \end{bmatrix}$$

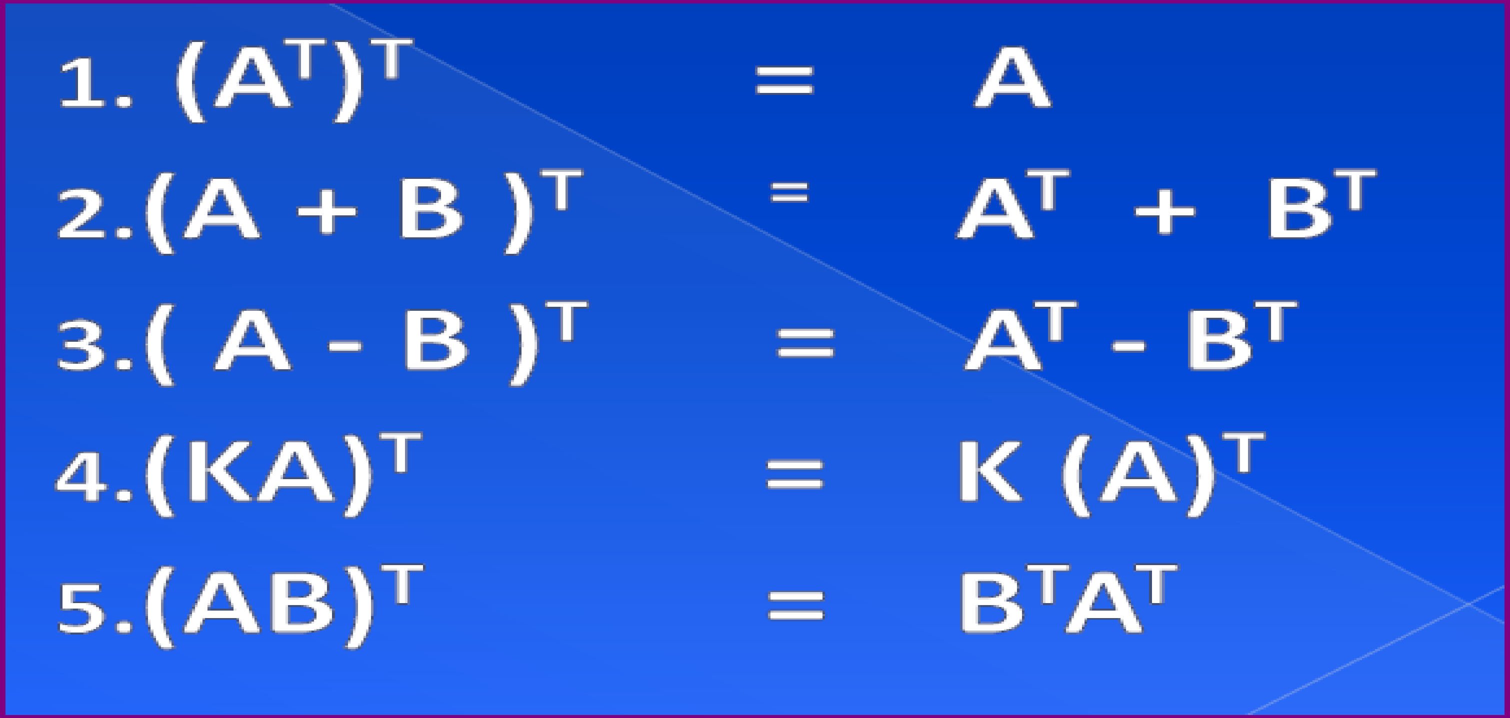
Transpose of a Matrix

$$\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

Input Matrix

Transpose Matrix

PROPERTIES OF TRANSPOSE OF A MATRIX



1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$
3. $(A - B)^T = A^T - B^T$
4. $(KA)^T = K(A)^T$
5. $(AB)^T = B^T A^T$

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SYMMETRIC AND SKEW SYMMETRIC MATRICES

A SQUARE MATRIX **A** IS SAID TO BE SYMMETRIC

IF $A' = A$

A SQUARE MATRIX **A** IS SAID TO BE
SKEW SYMMETRIC

IF $A' = -A$

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Eg

- Eg: A= $\begin{bmatrix} 1 & 5 & -8 \\ 5 & 3 & 0 \\ -8 & 0 & 6 \end{bmatrix}$

A^T = $\begin{bmatrix} 1 & 5 & -8 \\ 5 & 3 & 0 \\ -8 & 0 & 6 \end{bmatrix}$ SO, A is symmetric

Eg 2) B = $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$: B^T = $\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$ = - $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$ = -B

B is skew symmetric

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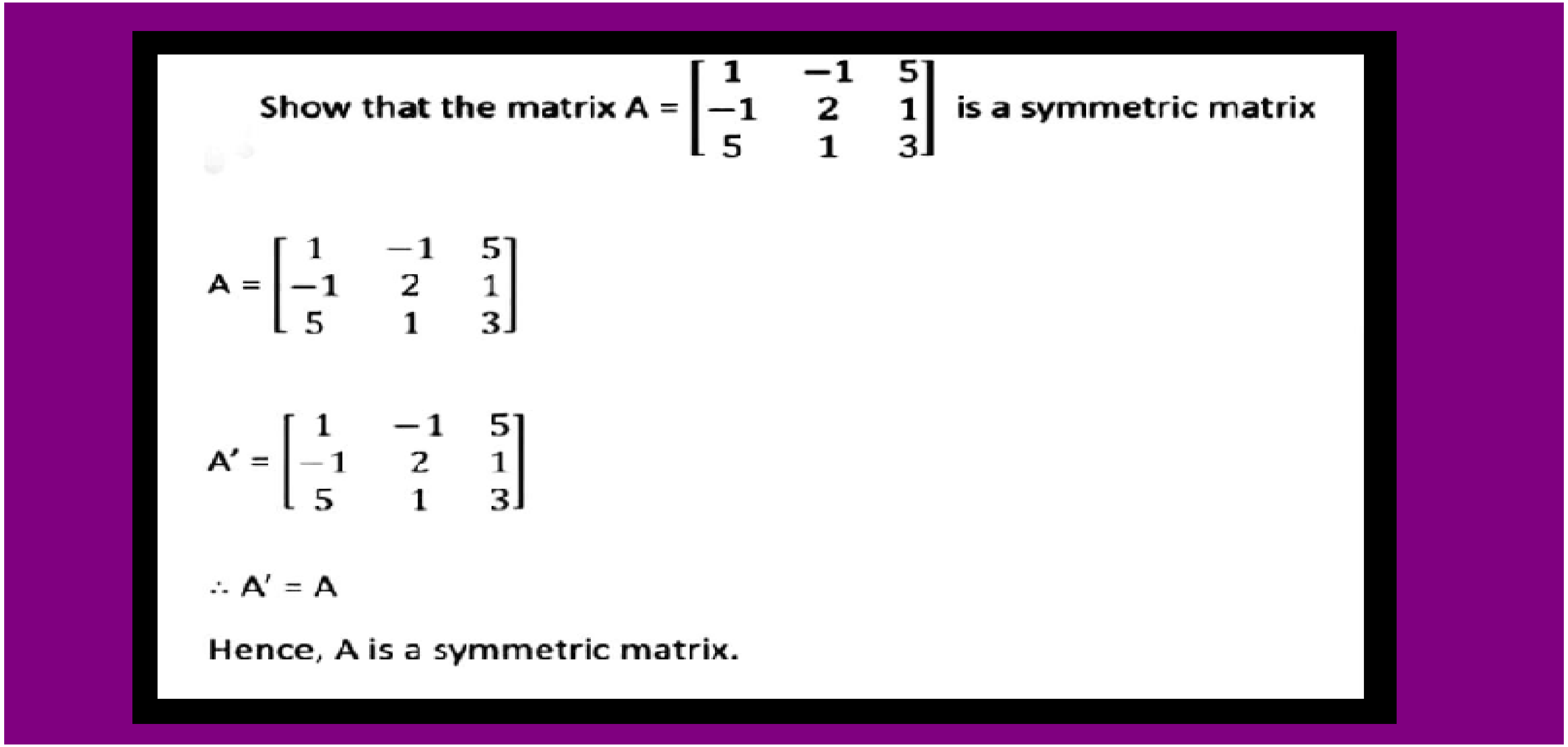
Eg:



Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

$\therefore A' = A$

Hence, A is a symmetric matrix.

Eg:

- A = $\begin{bmatrix} 0 & 4 & -5 \\ -4 & 0 & 1 \\ 5 & -1 & 0 \end{bmatrix}$ Show that it's a skew symmetric.

$$A^T = \begin{bmatrix} 0 & -4 & 5 \\ 4 & 0 & -1 \\ -5 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 4 & -5 \\ -4 & 0 & 1 \\ 5 & -1 & 0 \end{bmatrix} = -A$$

Hence A is Skew Symmetric

THEOREM 1

For any square matrix A With real numbers,A +$A^T$is a Symmetric matrix and A-$A^T$is a skew symmetric matrix

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Eg:

- Let A = $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \\ 4 & -5 & 2 \end{bmatrix}$ Show that A + A^T Symmetric and A - A^T skew symmetric.

A + A^T = $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \\ 4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & -5 \\ 0 & 5 & 2 \end{bmatrix}$ = $\begin{bmatrix} 2 & 5 & 4 \\ 5 & 2 & 0 \\ 4 & 0 & 4 \end{bmatrix}$

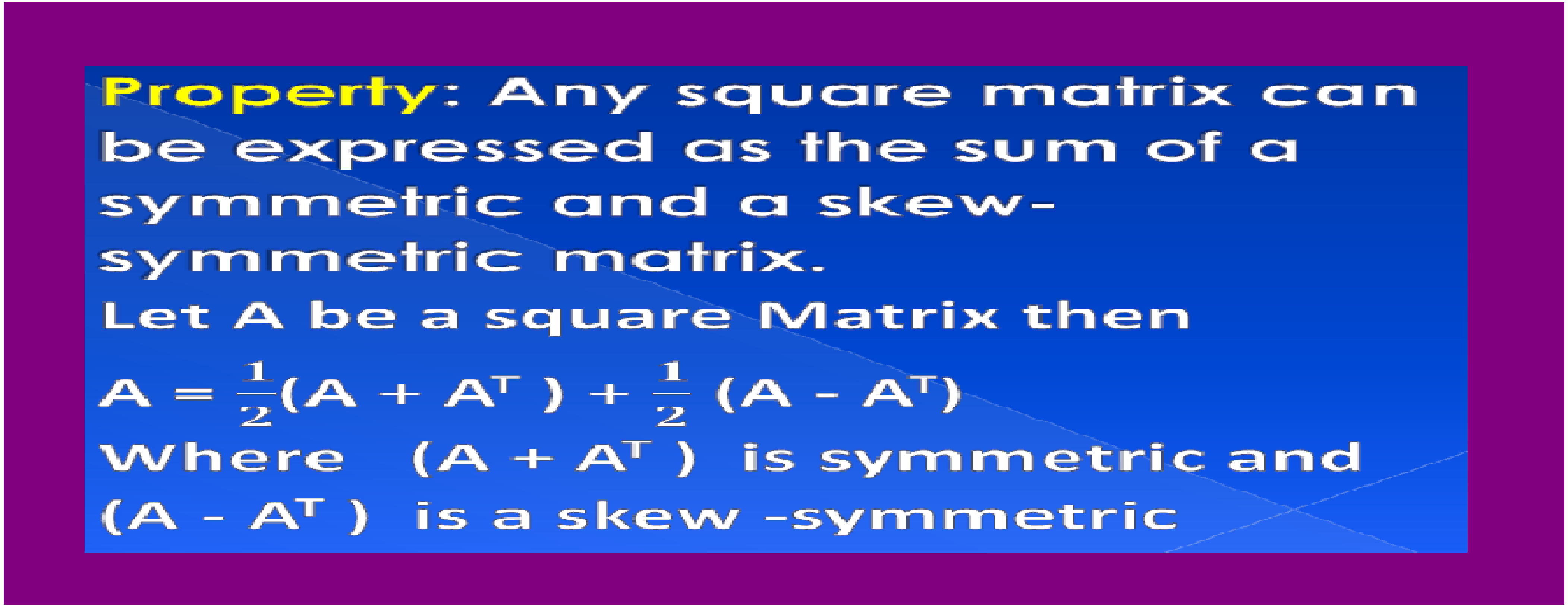
$(A + A^T)^T$ = $\begin{pmatrix} 2 & 5 & 4 \\ 5 & 2 & 0 \\ 4 & 0 & 4 \end{pmatrix}$ =A + A^T IS SYMMETRIC

A - A^T = $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \\ 4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & -5 \\ 0 & 5 & 2 \end{bmatrix}$ = $\begin{pmatrix} 0 & -1 & -4 \\ 1 & 0 & 10 \\ 4 & -10 & 0 \end{pmatrix}$

$(A - A^T)^T$ = $\begin{pmatrix} 0 & 1 & 4 \\ -1 & 0 & -10 \\ -4 & 10 & 0 \end{pmatrix}$ = - $\begin{pmatrix} 0 & -1 & -4 \\ 1 & 0 & 10 \\ 4 & -10 & 0 \end{pmatrix}$ = -(A - A^T)

(A - A^T) is skew symmetric

Theorem 2



Property: Any square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix.

Let A be a square Matrix then

$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

Where $(A + A^T)$ is symmetric and $(A - A^T)$ is a skew -symmetric

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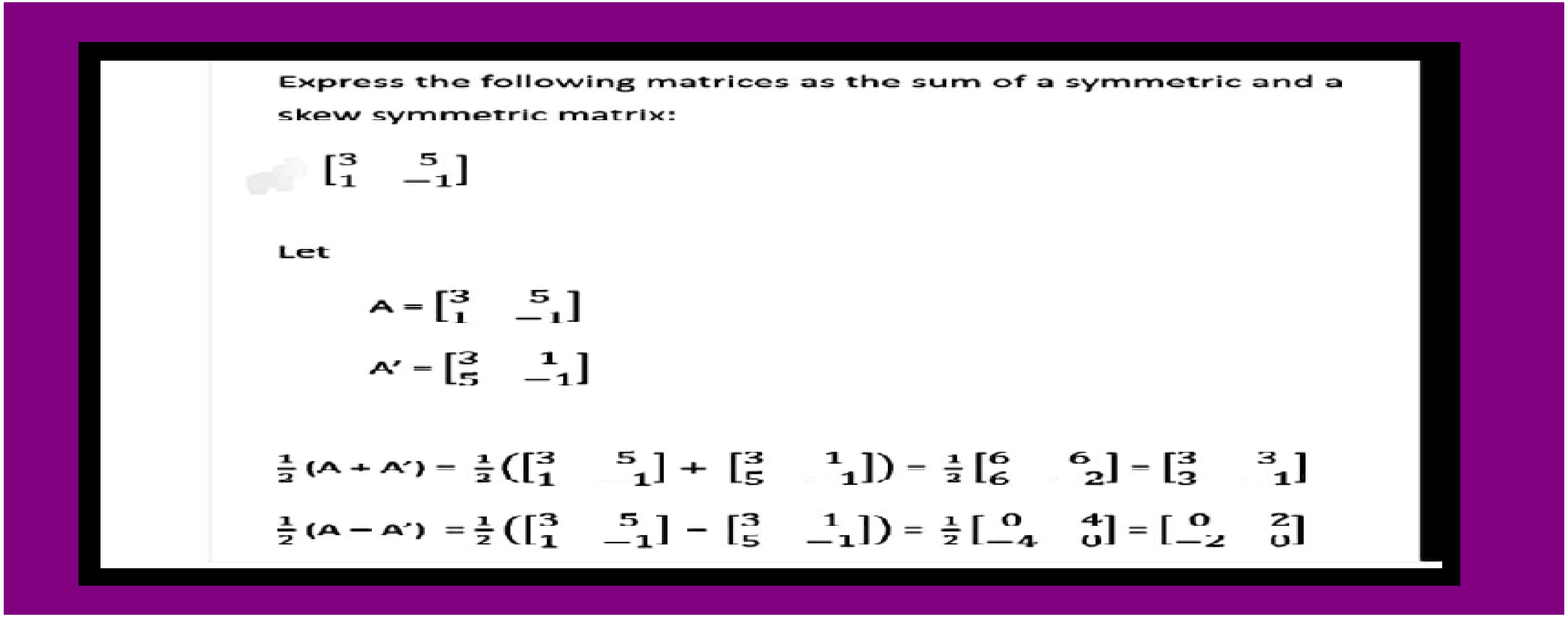
Eg:



Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

$$\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$\frac{1}{2}(A + A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & 1 \end{bmatrix}$$

$$\frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

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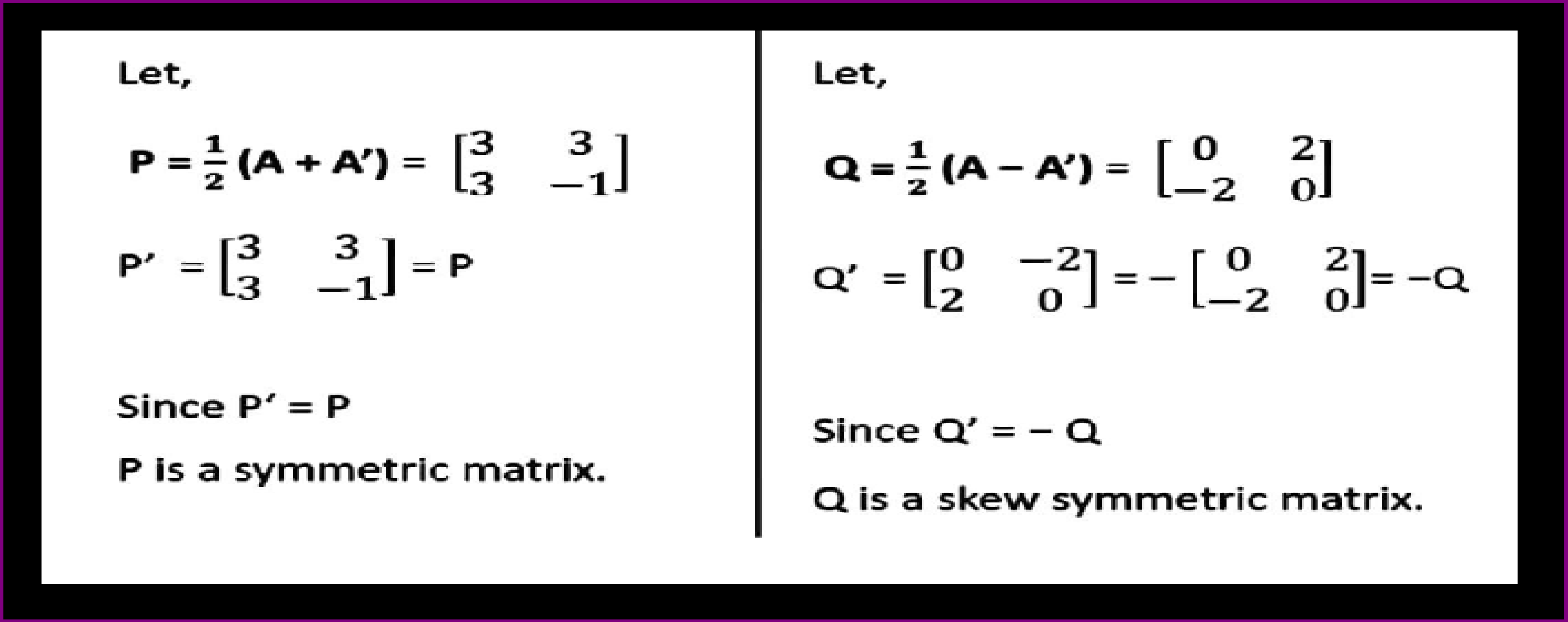

Let,

$$P = \frac{1}{2}(A + A') = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} = P$$

Since P' = P

P is a symmetric matrix.

Let,

$$Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q$$

Since Q' = − Q

Q is a skew symmetric matrix.

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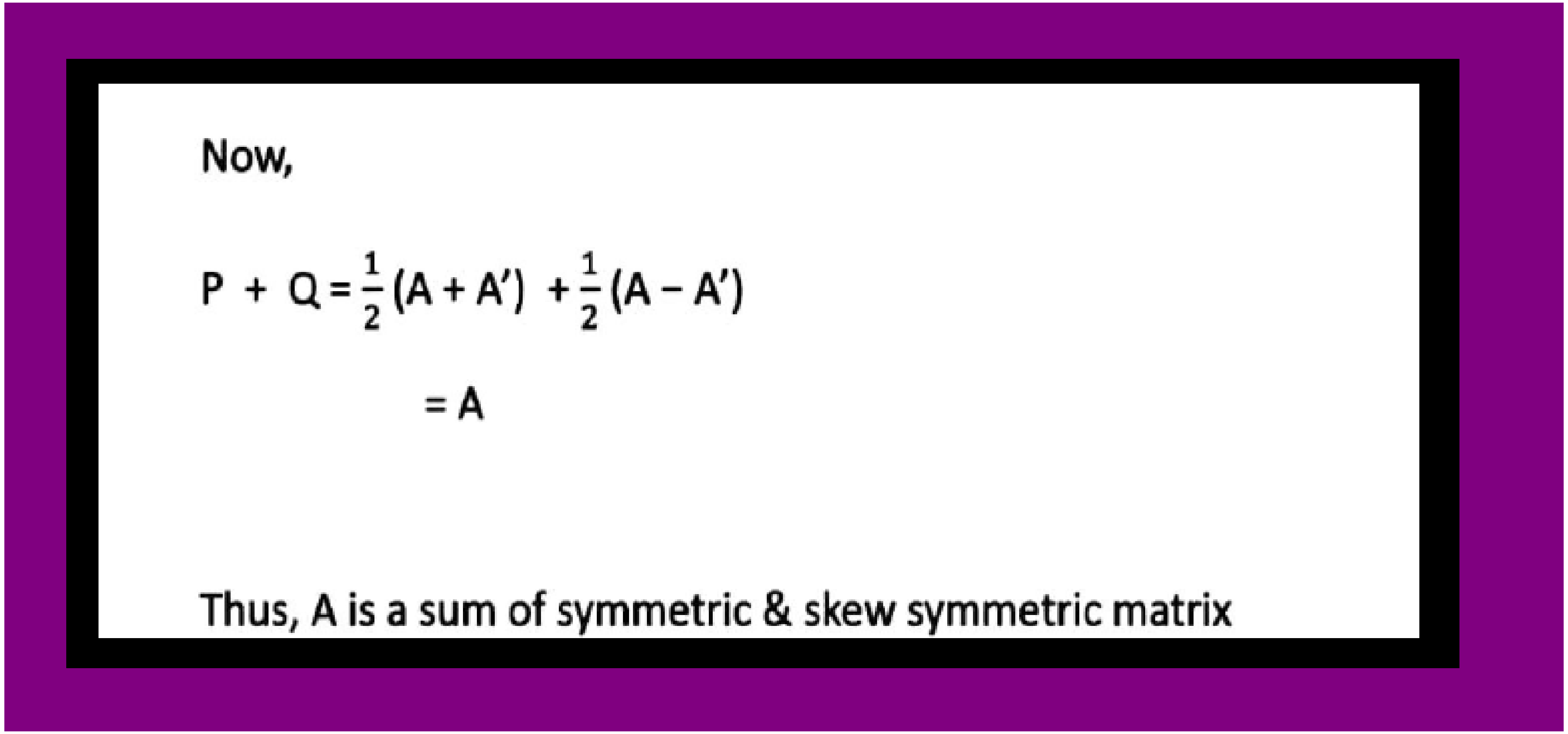

Now,

$$P + Q = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$= A$$

Thus, A is a sum of symmetric & skew symmetric matrix

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MATRICES PART – 4

ELEMENTARY OPERATIONS/ TRANSFORMATIONS ON A MATRIX

TESSY ROY VARGHESE

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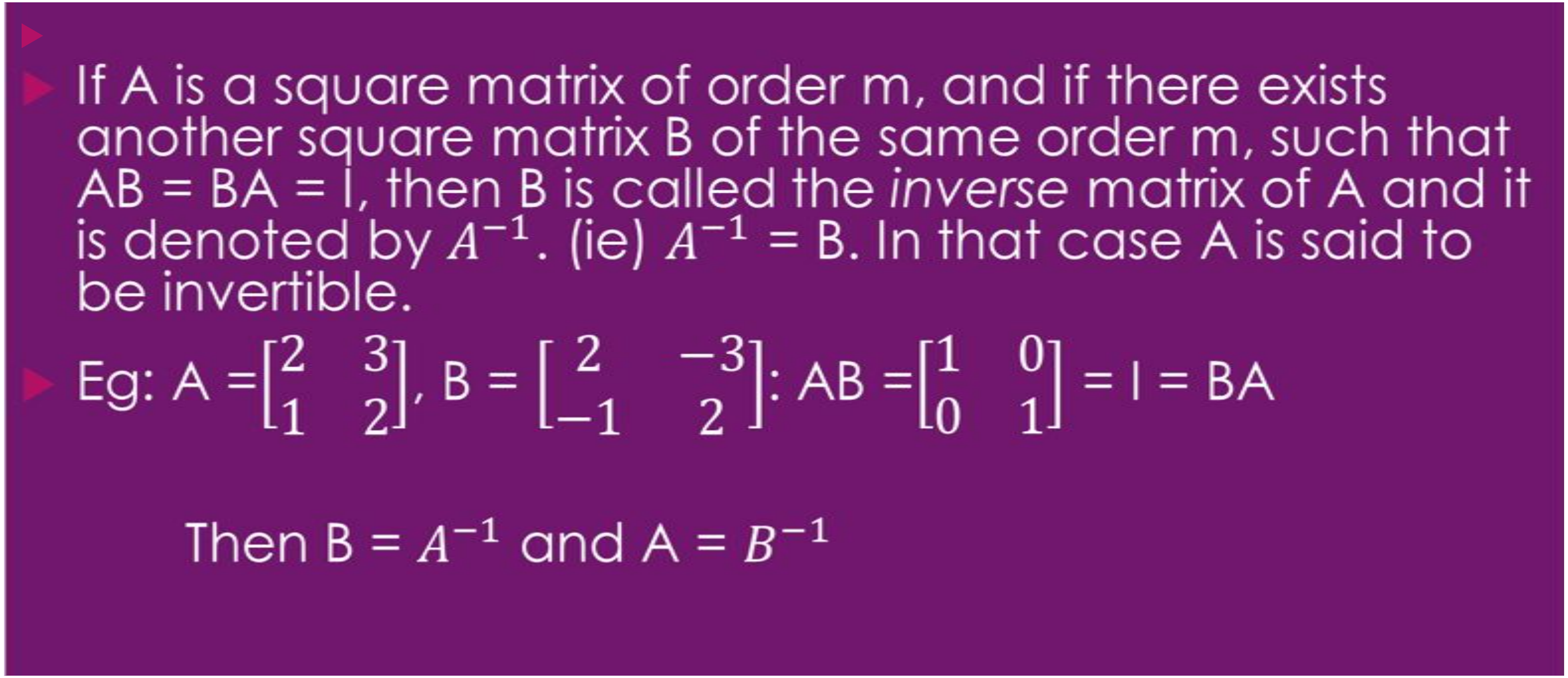
INVERTIBLE MATRICES



- If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the *inverse* matrix of A and it is denoted by A^{-1}. (ie) A^{-1} = B. In that case A is said to be invertible.
- Eg: A $=\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$: AB $=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = I = BA

Then B = A^{-1} and A = B^{-1}

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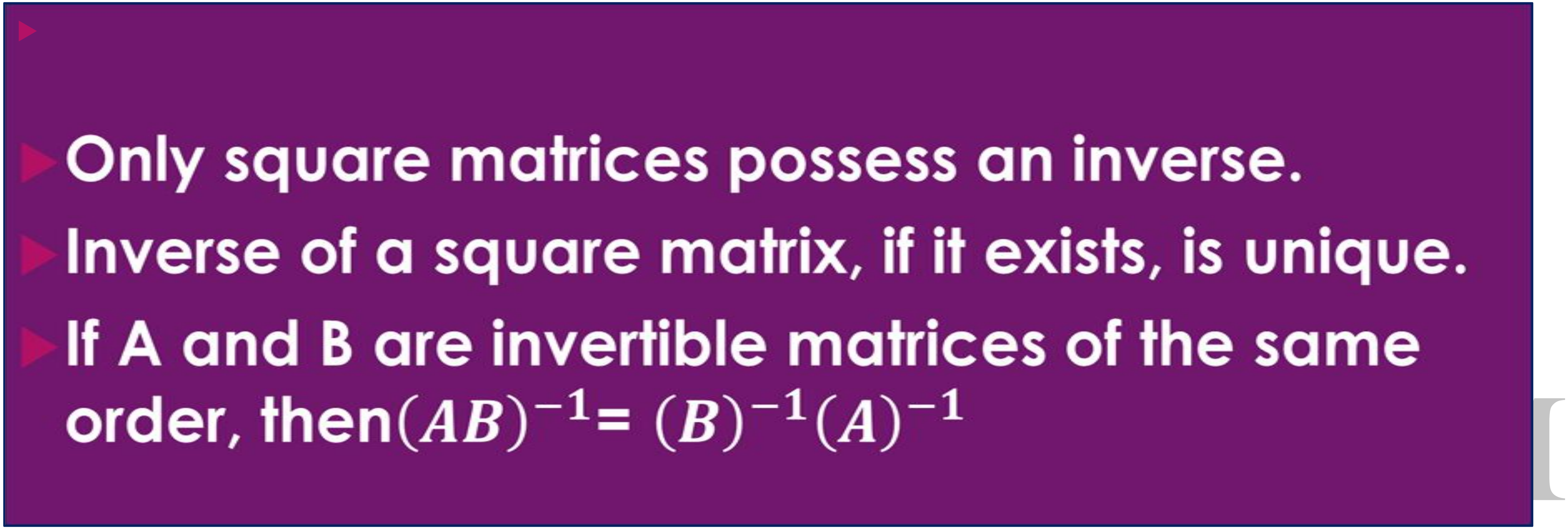
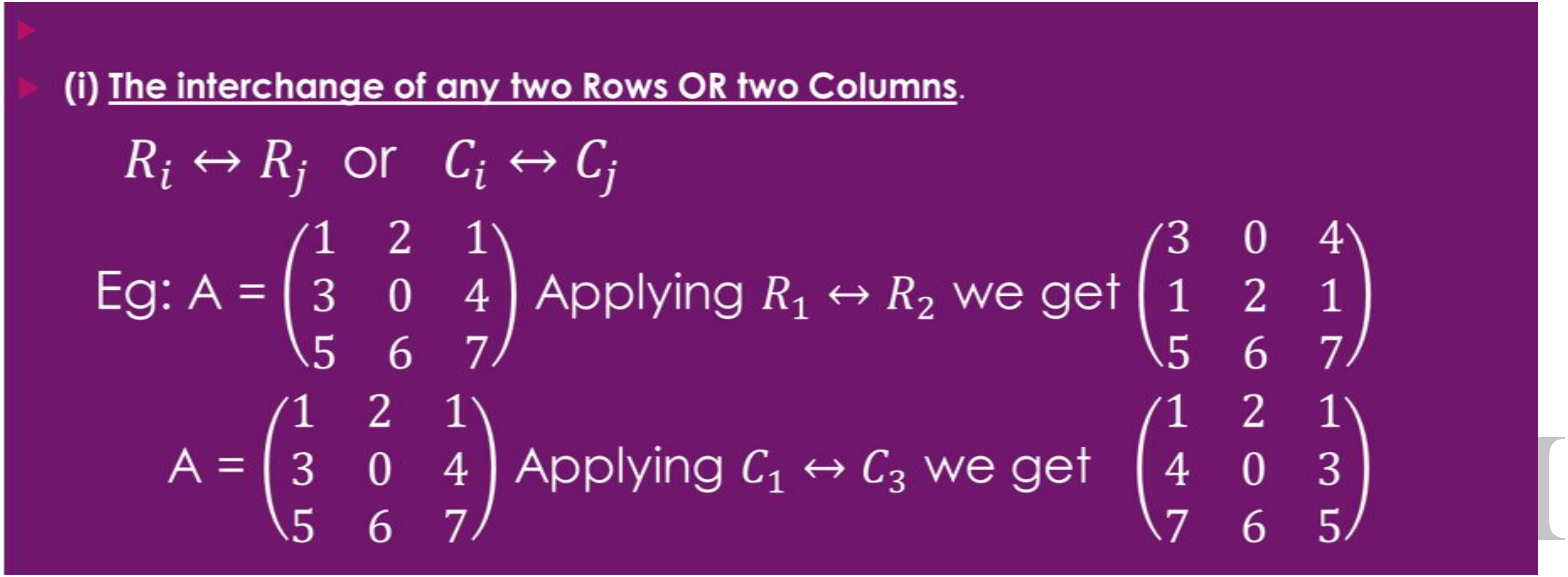
3 IMPORTANT NOTES



- Only square matrices possess an inverse.
- Inverse of a square matrix, if it exists, is unique.
- If A and B are invertible matrices of the same order, then $(AB)^{-1}= (B)^{-1}(A)^{-1}$

THREE ELEMENTARY OPERATIONS/ TRANSFORMATIONS



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- (i) <u>The interchange of any two Rows OR two Columns</u>.

$$R_i \leftrightarrow R_j \text{ or } C_i \leftrightarrow C_j$$

$$\text{Eg: } A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & 4 \\ 5 & 6 & 7 \end{pmatrix} \text{ Applying } R_1 \leftrightarrow R_2 \text{ we get } \begin{pmatrix} 3 & 0 & 4 \\ 1 & 2 & 1 \\ 5 & 6 & 7 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & 4 \\ 5 & 6 & 7 \end{pmatrix} \text{ Applying } C_1 \leftrightarrow C_3 \text{ we get } \begin{pmatrix} 1 & 2 & 1 \\ 4 & 0 & 3 \\ 7 & 6 & 5 \end{pmatrix}$$

ii)The multiplication of the elements of any row or column by a non zero number.

- $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$, $k \neq 0$
- Eg: A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ Applying $R_2 \rightarrow 3R_2$ we get $\begin{bmatrix} 1 & 2 & 3 \\ 12 & 15 & 18 \end{bmatrix}$
- A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ Applying $C_1 \rightarrow \frac{-1}{2} C_1$ we get $\begin{bmatrix} \frac{-1}{2} & 2 & 3 \\ -2 & 5 & 6 \end{bmatrix}$

(iii) The addition to the elements of any row or column , the corresponding elements of any other row or column multiplied by any non zero number.

- $R_i \to R_i + \text{k}\, R_j$ or $C_i \to C_i + \text{k}\, C_j$, $\text{k} \neq 0$

Eg: A = $\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ Applying $R_2 \to R_2 + 3\, R_1$, we get

$\begin{pmatrix} 1 & 0 \\ 2 + 3.1 & 3 + 3.0. \end{pmatrix}$ = $\begin{pmatrix} 1 & 0 \\ 5 & 3. \end{pmatrix}$

A = $\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ Applying $C_1 \to C_1 +$ (-1) C_2 , we get $\begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$

STEPS TO FIND INVERSE BY ROW TRANSFORMATION

- ↓1.Write A = **I**A, where **I** is the identity matrix.
- 2.Using a sequence of elementary row operations, reduce LHS **A to I** and perform similar operations **on I on RHS**.

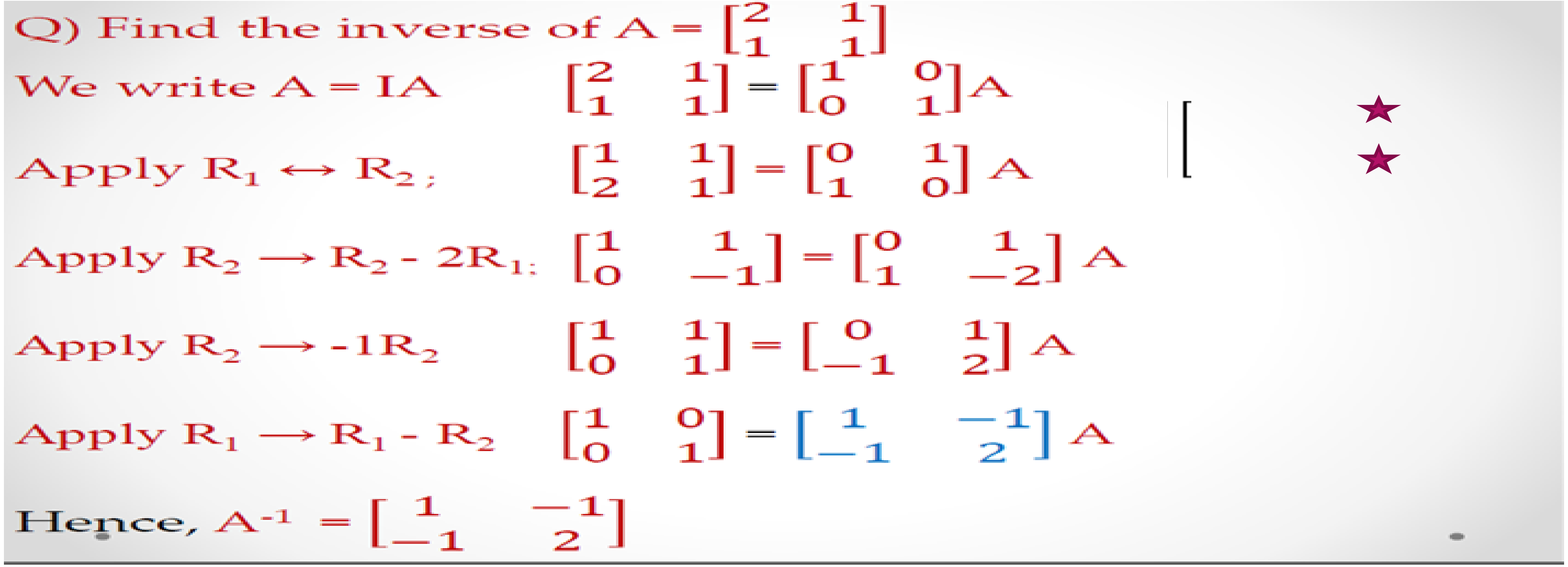
Eg: If A = $\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\downarrow \qquad\qquad \downarrow$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{pmatrix} \mathbf{A} \longrightarrow \mathbf{I = BA} \longrightarrow \mathbf{B} = (A)^{-1}$$

To find the inverse of A with order 2



Q) Find the inverse of $A = \begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}$

We write $A = IA$ $\quad \begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$

Apply $R_1 \leftrightarrow R_2$; $\quad \begin{bmatrix}1 & 1\\ 2 & 1\end{bmatrix} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} A$

Apply $R_2 \longrightarrow R_2 - 2R_1$: $\quad \begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix} = \begin{bmatrix}0 & 1\\ 1 & -2\end{bmatrix} A$

Apply $R_2 \longrightarrow -1R_2$ $\quad \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & 2\end{bmatrix} A$

Apply $R_1 \longrightarrow R_1 - R_2$ $\quad \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}$

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HOW TO VERIFY INVERSE OF ORDER 2 MATRIX

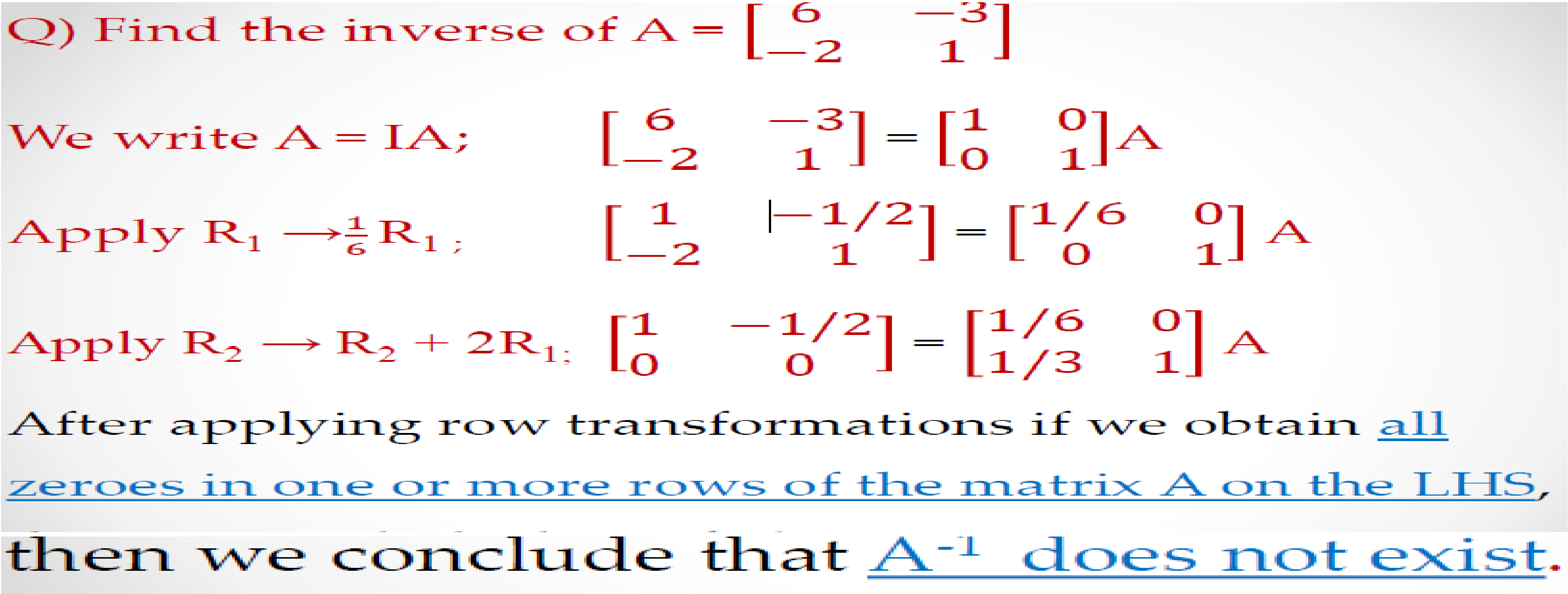
- $$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} ;$$

$$A^{-1} = \frac{1}{12-10}\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$$

A^{-1} Does not exist



Q) Find the inverse of $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

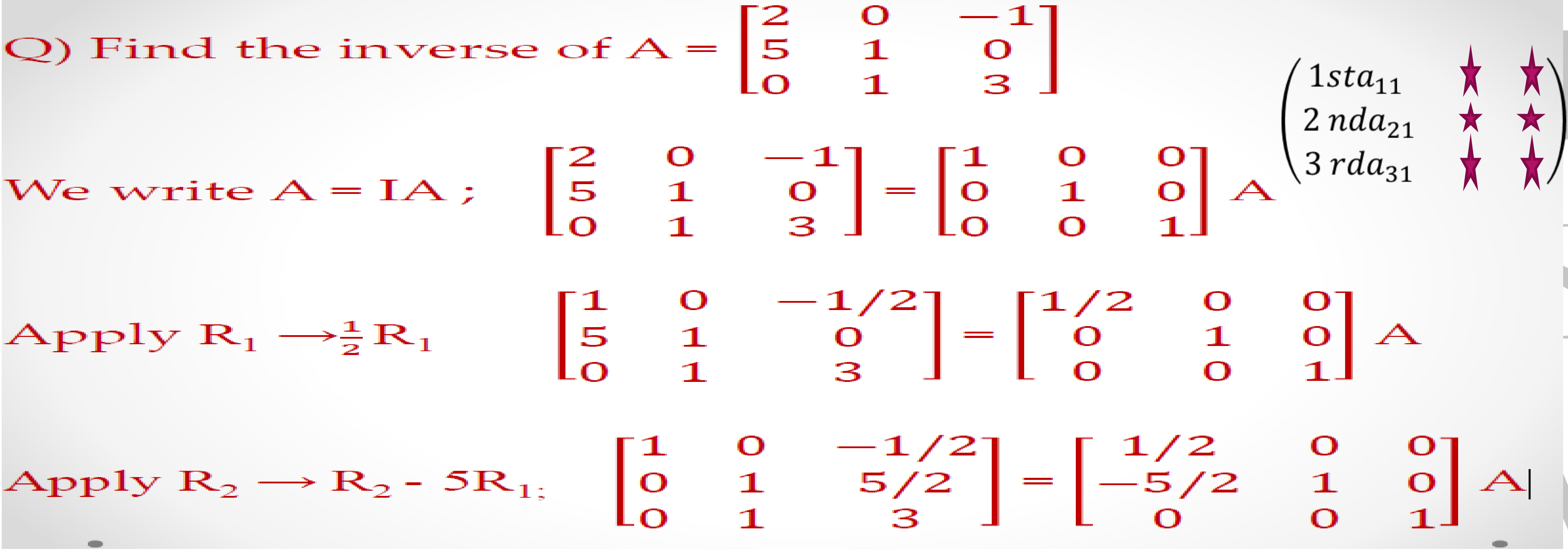
We write A = IA; $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Apply $R_1 \rightarrow \frac{1}{6} R_1$; $\begin{bmatrix} 1 & -1/2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1/6 & 0 \\ 0 & 1 \end{bmatrix} A$

Apply $R_2 \rightarrow R_2 + 2R_1$; $\begin{bmatrix} 1 & -1/2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1/6 & 0 \\ 1/3 & 1 \end{bmatrix} A$

After applying row transformations if we obtain all zeroes in one or more rows of the matrix A on the LHS, then we conclude that A^{-1} does not exist.

To find the inverse of A with order 3



Q) Find the inverse of A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$\begin{pmatrix} 1st\, a_{11} & ★ & ★ \\ 2\, nd\, a_{21} & ★ & ★ \\ 3\, rd\, a_{31} & ★ & ★ \end{pmatrix}$

We write A = IA ; $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Apply $R_1 \longrightarrow \frac{1}{2} R_1$ $\begin{bmatrix} 1 & 0 & -1/2 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Apply $R_2 \longrightarrow R_2 - 5R_1;$ $\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 5/2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ -5/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

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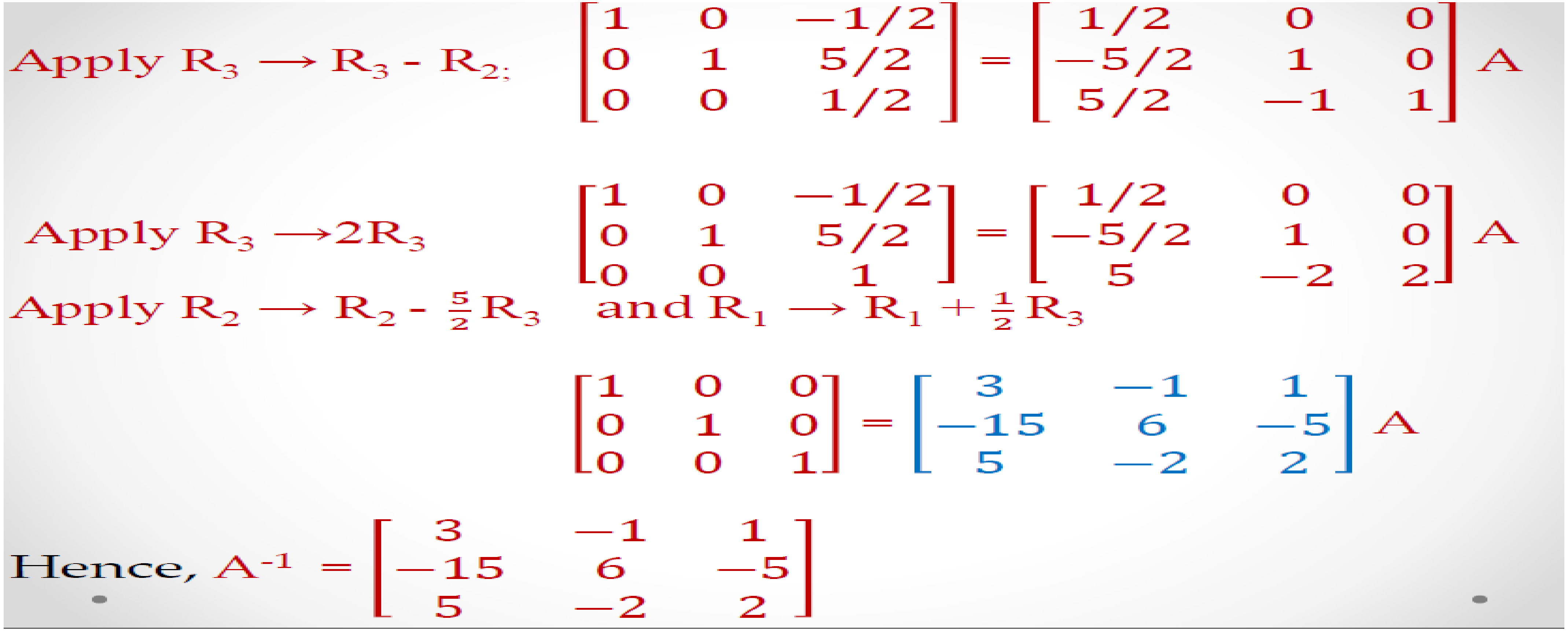

Apply $R_3 \longrightarrow R_3 - R_2$; $\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 5/2 \\ 0 & 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ -5/2 & 1 & 0 \\ 5/2 & -1 & 1 \end{bmatrix} A$

Apply $R_3 \longrightarrow 2R_3$ $\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 5/2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ -5/2 & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$

Apply $R_2 \longrightarrow R_2 - \frac{5}{2}R_3$ and $R_1 \longrightarrow R_1 + \frac{1}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

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MATRIX- PART 5

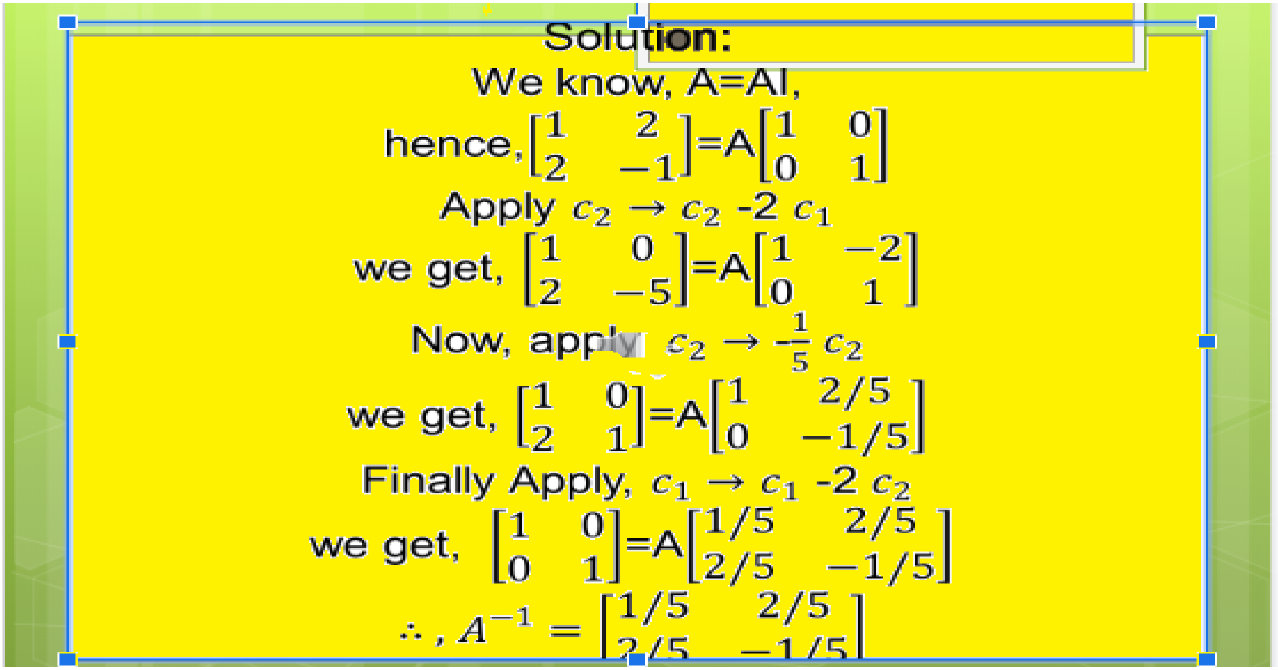
TESSY ROY VARGHESE

INDIAN SCHOOL MUSCAT

COLUMN TRANSFORMATION

- In Order to use elementary column operations, we write A = AI

- Eg: A = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ Find A^{-1} by column operations



Solution:

We know, A=AI,

hence, $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$=A$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Apply $c_2 \rightarrow c_2$ -2 c_1

we get, $\begin{bmatrix} 1 & 0 \\ 2 & -5 \end{bmatrix}$=A$\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

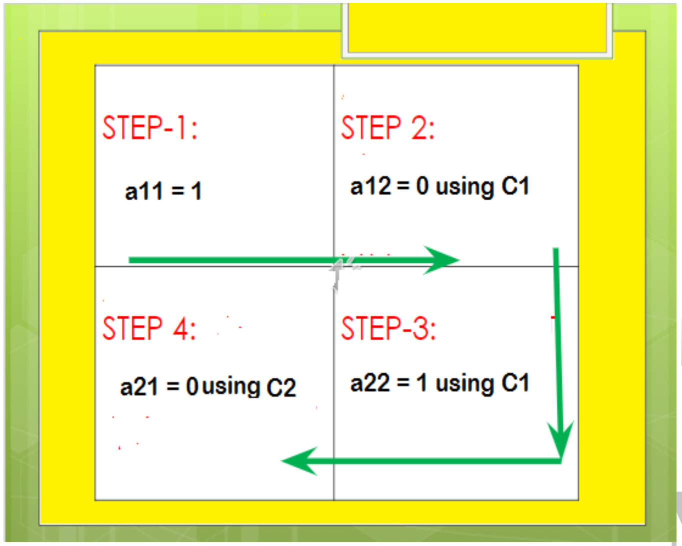
Now, apply $c_2 \rightarrow -\frac{1}{5} c_2$

we get, $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$=A$\begin{bmatrix} 1 & 2/5 \\ 0 & -1/5 \end{bmatrix}$

Finally Apply, $c_1 \rightarrow c_1$ -2 c_2

we get, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$=A$\begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$

$\therefore, A^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$



STEP-1:
a11 = 1

STEP 2:
a12 = 0 using C1

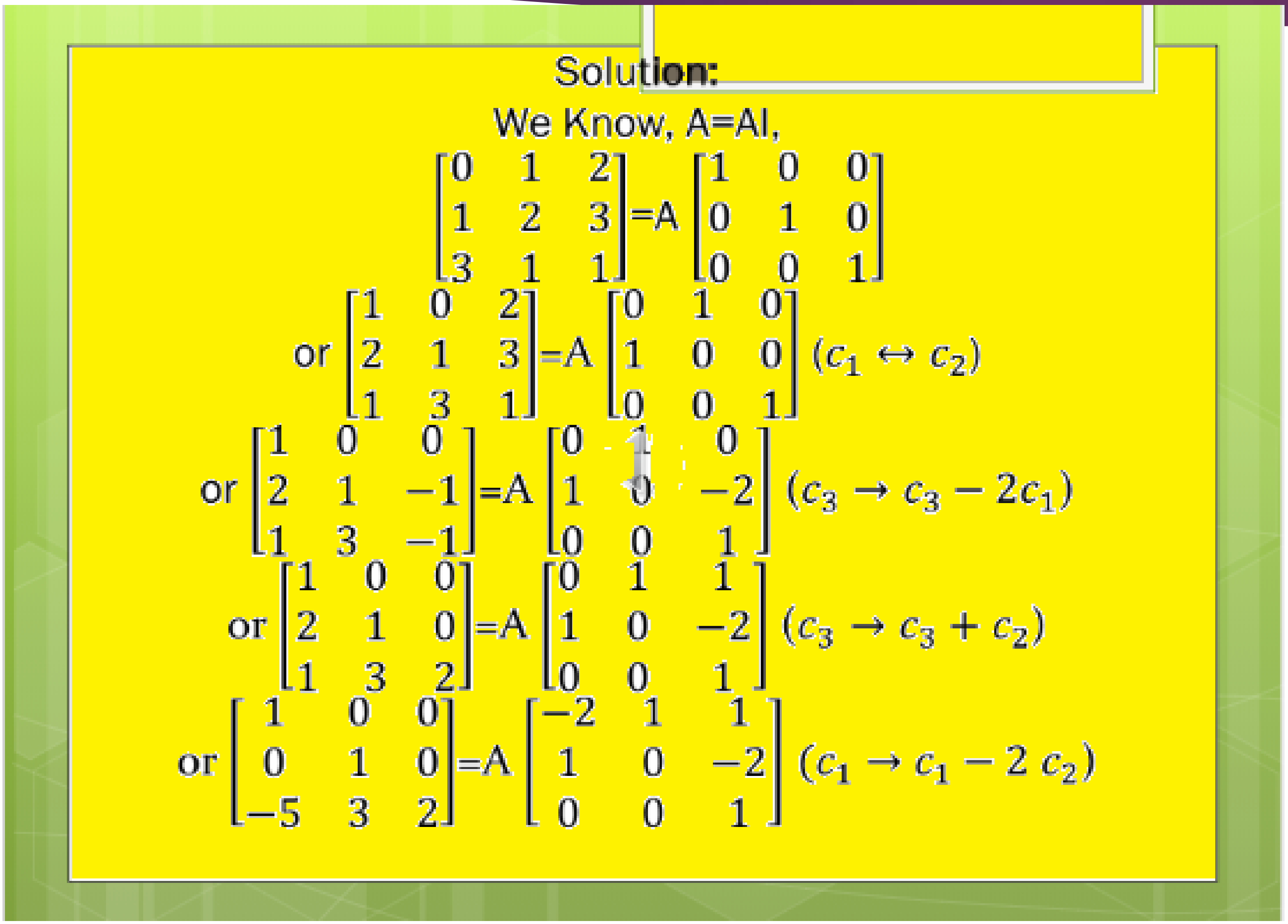
STEP-3:
a22 = 1 using C1

STEP 4:
a21 = 0using C2

Find inverse by column operation of matrix order 3

Q : $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

A = A I



Solution:

We Know, A=AI,

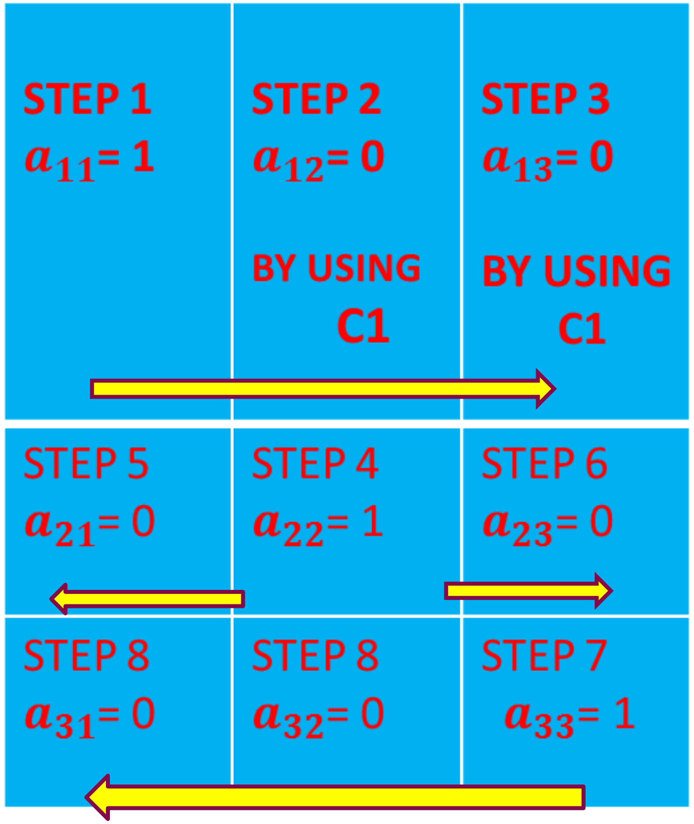
$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 1 & 3 & 1 \end{bmatrix} = A \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} (c_1 \leftrightarrow c_2)$$

$$\text{or } \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -1 \\ 1 & 3 & -1 \end{bmatrix} = A \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} (c_3 \rightarrow c_3 - 2c_1)$$

$$\text{or } \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 3 & 2 \end{bmatrix} = A \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} (c_3 \rightarrow c_3 + c_2)$$

$$\text{or } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 3 & 2 \end{bmatrix} = A \begin{bmatrix} -2 & 1 & 1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} (c_1 \rightarrow c_1 - 2\ c_2)$$



STEP 1 a_{11}= 1	**STEP 2** a_{12}= 0 BY USING C1	**STEP 3** a_{13}= 0 BY USING C1
STEP 5 a_{21}= 0	STEP 4 a_{22}= 1	STEP 6 a_{23}= 0
STEP 8 a_{31}= 0	STEP 8 a_{32}= 0	STEP 7 a_{33}= 1

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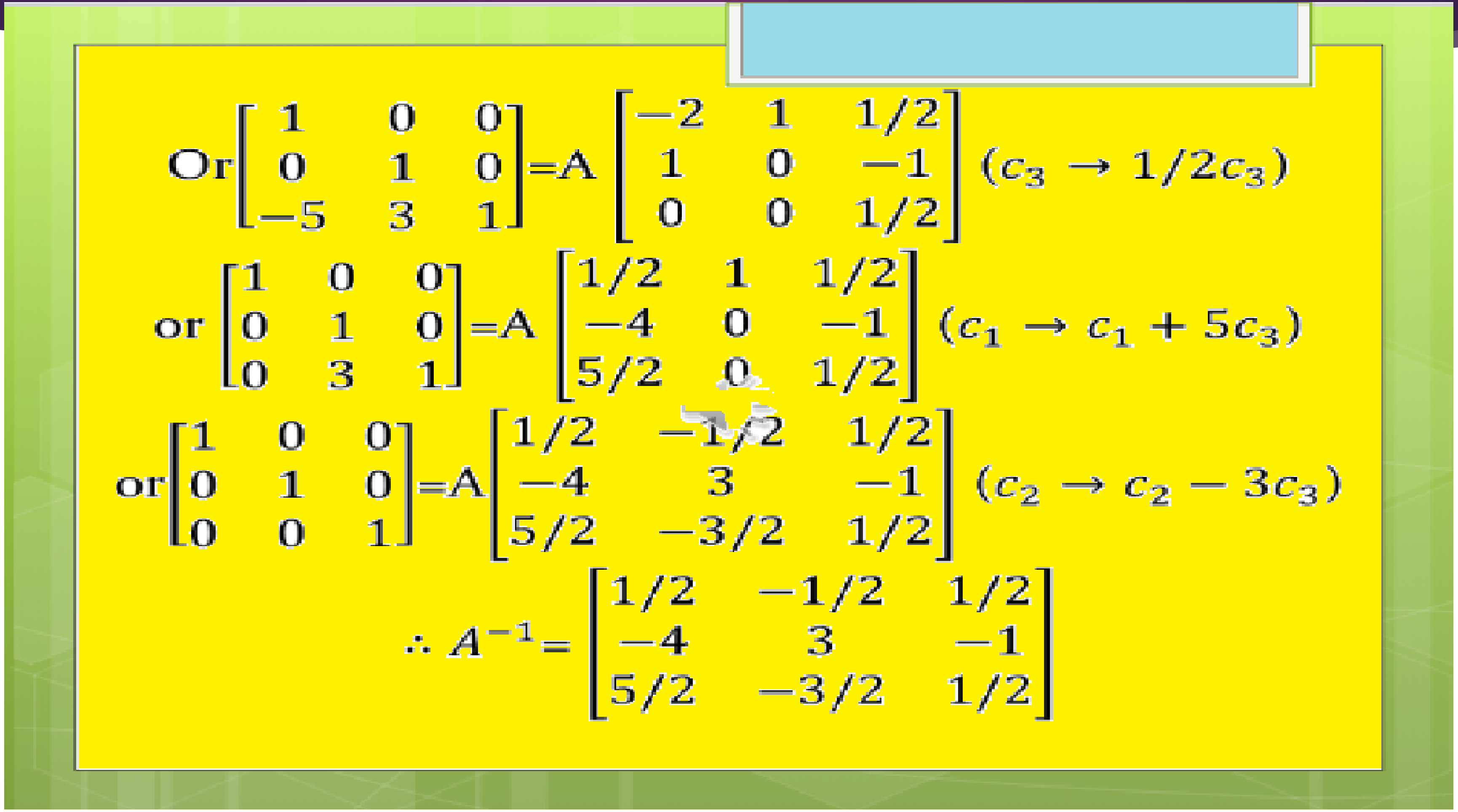

$$\text{Or}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ -5 & 3 & 1\end{bmatrix}=\text{A}\begin{bmatrix}-2 & 1 & 1/2\\ 1 & 0 & -1\\ 0 & 0 & 1/2\end{bmatrix}(c_3 \rightarrow 1/2c_3)$$

$$\text{or}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 3 & 1\end{bmatrix}=\text{A}\begin{bmatrix}1/2 & 1 & 1/2\\ -4 & 0 & -1\\ 5/2 & 0 & 1/2\end{bmatrix}(c_1 \rightarrow c_1 + 5c_3)$$

$$\text{or}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\text{A}\begin{bmatrix}1/2 & -1/2 & 1/2\\ -4 & 3 & -1\\ 5/2 & -3/2 & 1/2\end{bmatrix}(c_2 \rightarrow c_2 - 3c_3)$$

$$\therefore A^{-1}=\begin{bmatrix}1/2 & -1/2 & 1/2\\ -4 & 3 & -1\\ 5/2 & -3/2 & 1/2\end{bmatrix}$$

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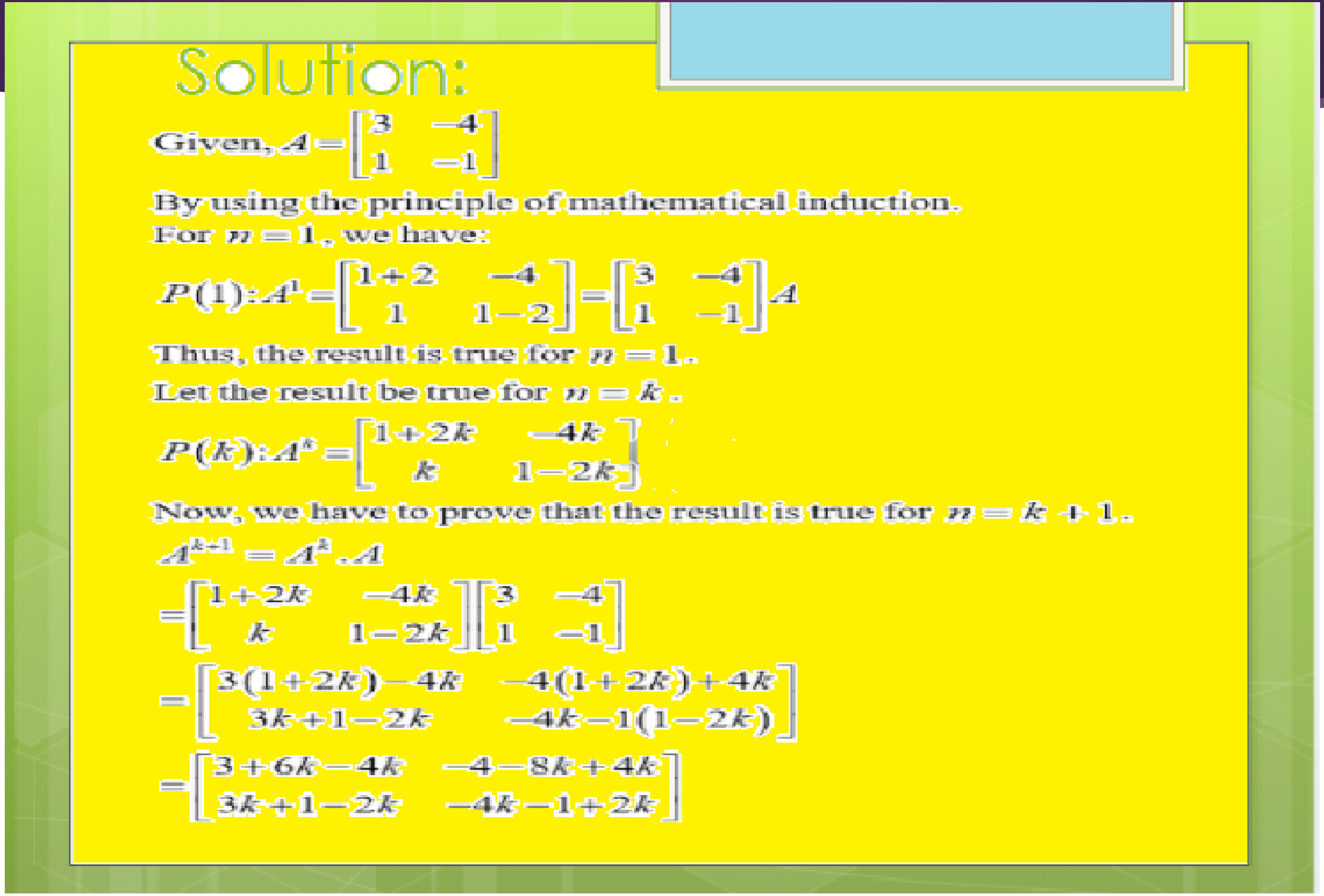
MIS.EXERCISE



Solution:

Given, $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$

By using the principle of mathematical induction.

For $n=1$, we have:

$$P(1):A^1=\begin{bmatrix}1+2 & -4\\ 1 & 1-2\end{bmatrix}=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}A$$

Thus, the result is true for $n=1$.

Let the result be true for $n=k$.

$$P(k):A^k=\begin{bmatrix}1+2k & -4k\\ k & 1-2k\end{bmatrix}$$

Now, we have to prove that the result is true for $n=k+1$.

$$A^{k+1}=A^k.A$$

$$=\begin{bmatrix}1+2k & -4k\\ k & 1-2k\end{bmatrix}\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}3(1+2k)-4k & -4(1+2k)+4k\\ 3k+1-2k & -4k-1(1-2k)\end{bmatrix}$$

$$=\begin{bmatrix}3+6k-4k & -4-8k+4k\\ 3k+1-2k & -4k-1+2k\end{bmatrix}$$

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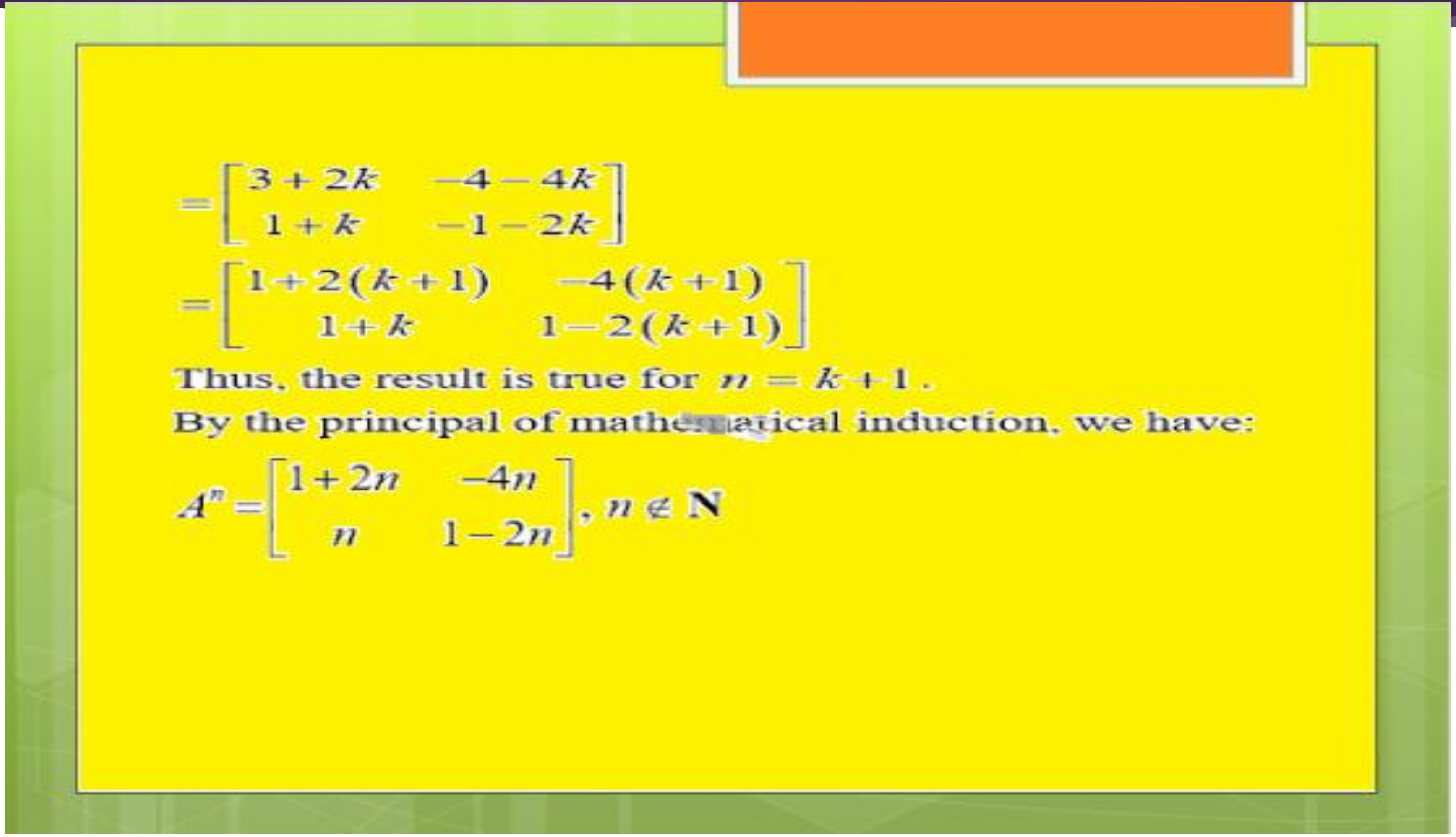

$$=\begin{bmatrix}3+2k & -4-4k\\ 1+k & -1-2k\end{bmatrix}$$

$$=\begin{bmatrix}1+2(k+1) & -4(k+1)\\ 1+k & 1-2(k+1)\end{bmatrix}$$

Thus, the result is true for $n=k+1$.

By the principal of mathematical induction, we have:

$$A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}, n \notin \mathbf{N}$$

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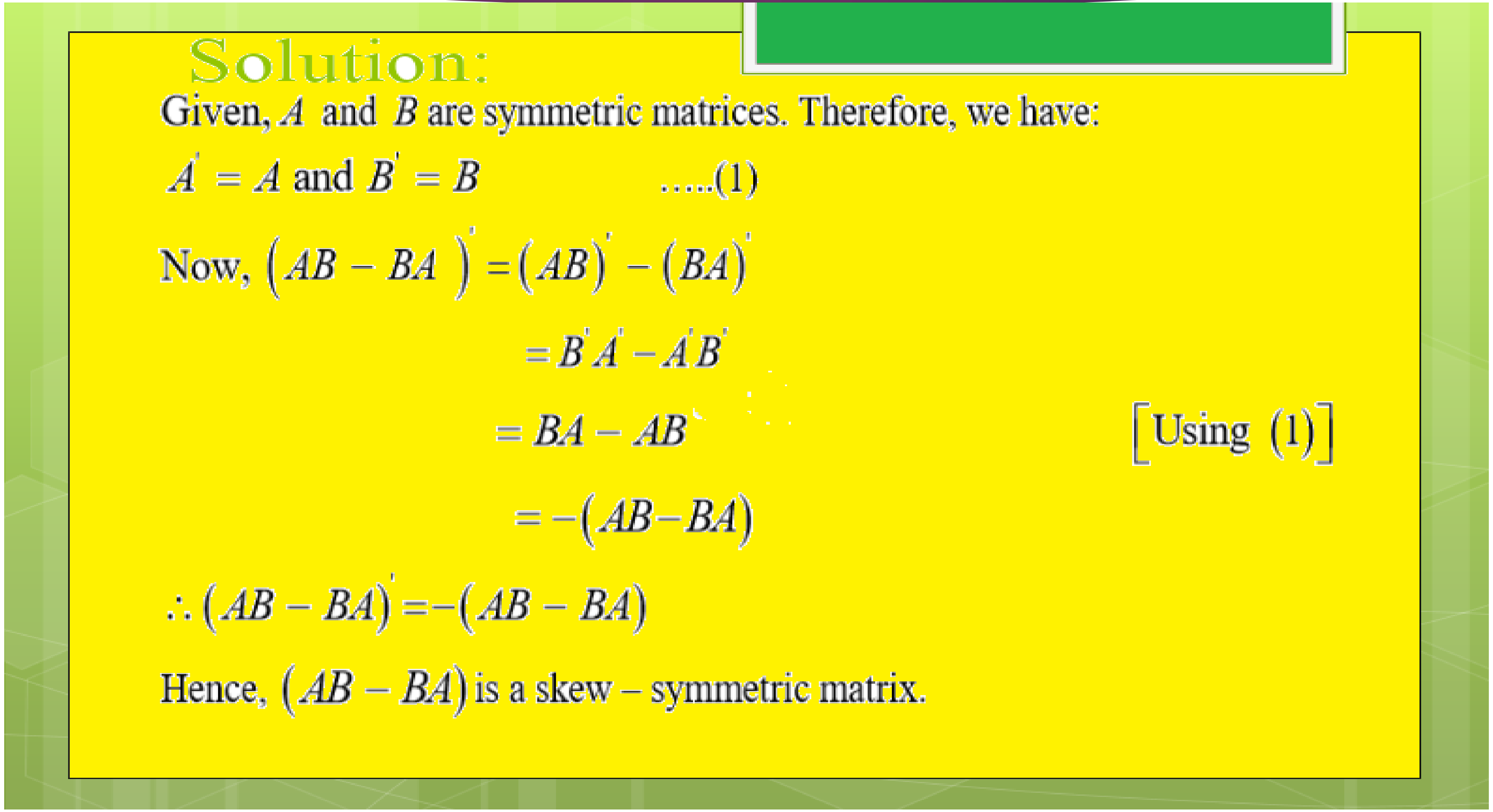
Q 2.If A and B are symmetric matrices AB-BA is a skew symmetric matrix



Solution:

Given, A and B are symmetric matrices. Therefore, we have:

$A' = A$ and $B' = B$ .....(1)

Now, $(AB - BA)' = (AB)' - (BA)'$

$$= B'A' - A'B'$$

$$= BA - AB \qquad [\text{Using } (1)]$$

$$= -(AB - BA)$$

$\therefore (AB - BA)' = -(AB - BA)$

Hence, $(AB - BA)$ is a skew – symmetric matrix.

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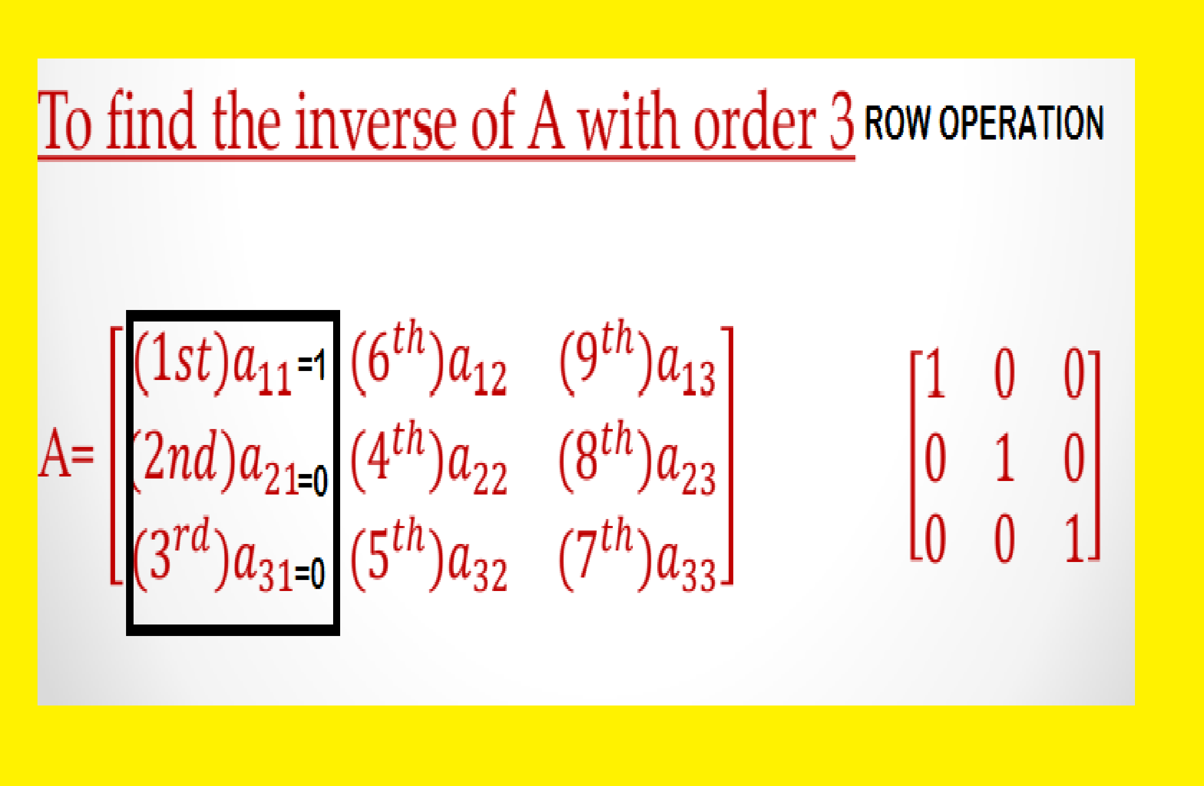
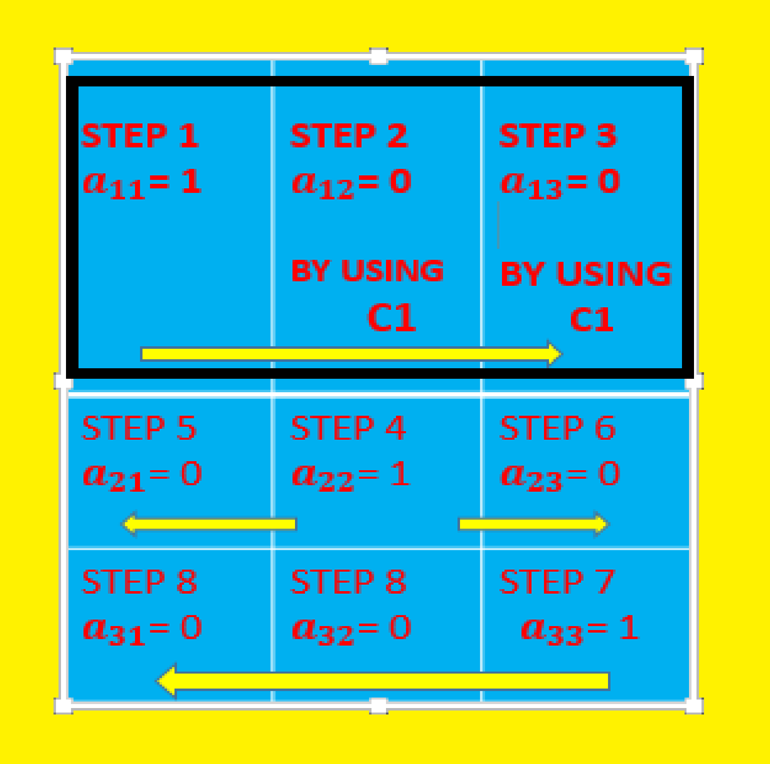
Rules



To find the inverse of A with order 3 ROW OPERATION

$$A=\begin{bmatrix} (1st)a_{11}=1 & (6^{th})a_{12} & (9^{th})a_{13} \\ (2nd)a_{21}=0 & (4^{th})a_{22} & (8^{th})a_{23} \\ (3^{rd})a_{31}=0 & (5^{th})a_{32} & (7^{th})a_{33} \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



STEP 1 $a_{11}=1$	**STEP 2** $a_{12}=0$ **BY USING C1**	**STEP 3** $a_{13}=0$ **BY USING C1**
STEP 5 $a_{21}=0$	STEP 4 $a_{22}=1$	STEP 6 $a_{23}=0$
STEP 8 $a_{31}=0$	STEP 8 $a_{32}=0$	STEP 7 $a_{33}=1$

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